$\mu(x) = \mu_o(1 + ax)$ and $\lambda(x) = k\mu(x)$, where μ_o, a and k are constants. For such a material, show that in the absence of body forces the two-dimensional Navier's equations become

$$\mu_o(1 + ax)\left(k\frac{\partial \vartheta}{\partial x} + 2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y}\right) + \mu_o a\left(k\vartheta + 2\frac{\partial u}{\partial x}\right) = 0$$
$$\mu_o(1 + ax)\left(k\frac{\partial \vartheta}{\partial y} + 2\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y}\right) + \mu_o a\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 0$$

where $\vartheta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$.

5-7. Carry out the integration details to develop the displacements (5.7.4) in Example 5-1.

5-8. Using the inverse method, investigate which problem may be solved by the two-dimensional stress distribution $\sigma_x = Axy,\ \tau_{xy} = B + Cy^2,\ \sigma_y = 0$, where A, B, and C are constants. First, show that the equilibrium equations require $C = -A/2$. Next choose a rectangular domain $0 \geq x \geq l$ and $-h \geq y \geq h$ and show that these stresses could represent the solution to a cantilever beam under end loading.

5-9. Show that the following stress components satisfy the equations of equilibrium with zero body forces, but are not the solution to a problem in elasticity:

$$\sigma_x = c[y^2 + v(x^2 - y^2)]$$
$$\sigma_y = c[x^2 + v(y^2 - x^2)]$$
$$\sigma_z = cv(x^2 + y^2)$$
$$\tau_{xy} = -2cvxy$$
$$\tau_{yz} = \tau_{zx} = 0,\ c = \text{constant} \neq 0$$

5-10*. Consider the problem of a concentrated force acting normal to the free surface of a semi-infinite solid as shown in case (a) of the following figure. The two-dimensional stress field for this problem is given by equations (8.4.36) as

$$\sigma_x = -\frac{2Px^2y}{\pi(x^2 + y^2)^2}$$
$$\sigma_y = -\frac{2Py^3}{\pi(x^2 + y^2)^2}$$
$$\tau_{xy} = -\frac{2Pxy^2}{\pi(x^2 + y^2)^2}$$

Using this solution with the method of superposition, solve the problem with two concentrated forces as shown in case (b). Because problems (a) and (b) have the same resultant boundary loading, explicitly show that at distances far away from the loading