4-7. The displacements in an elastic material are given by

$$u = -\frac{M(1-v^2)}{EI}xy,\ v = \frac{M(1+v)v}{2EI}y^2 + \frac{M(1-v^2)}{2EI}(x^2 - \frac{l^2}{4}),\ w = 0$$

where M, E, I, and l are constant parameters. Determine the corresponding strain and stress fields and show that this problem represents the pure bending of a rectangular beam in the x,y plane.

4-8. If the elastic constants E, k, and μ are required to be positive, show that Poisson's ratio must satisfy the inequality $-1 < v < \frac{1}{2}$. For most real materials it has been found that $0 < v < \frac{1}{2}$. Show that this more restrictive inequality in this problem implies that $\lambda > 0$.

4-9. Under the condition that E is positive and bounded, determine the elastic moduli λ, μ, and k for the special cases of Poisson's ratio: $v = 0,\ \frac{1}{4},\ \frac{1}{2}$.

4-10. Show that Hooke's law for an isotropic material may be expressed in terms of spherical and deviatoric tensors by the two relations

$$\tilde{\sigma}_{ij} = 3k\tilde{e}_{ij},\ \hat{\sigma}_{ij} = 2\mu\hat{e}_{ij}$$

4-11. A sample is subjected to a test under *plane stress* conditions (specified by $\sigma_z = \tau_{zx} = \tau_{zy} = 0$) using a special loading frame that maintains an in-plane loading constraint $\sigma_x = 2\sigma_y$. Determine the slope of the stress-strain response σ_x vs. e_x for this sample.

4-12. A rectangular steel plate (thickness 4 mm) is subjected to a uniform biaxial stress field as shown in the following figure. Assuming all fields are uniform, determine changes in the dimensions of the plate under this loading.