Example 3.11-1

Determine directly the normal and shear components, σ_N and σ_{oct}, on the octahedral plane for the state of stress in Example 3.6-1, and verify the result for σ_{oct} by Eq. 3.11-4.

Solution

From Example 3.6-1, the stress vector on the octahedral plane is given by the matrix product

$$\begin{bmatrix} 75 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 25 \end{bmatrix} \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} = \begin{bmatrix} 75/\sqrt{3} \\ 50/\sqrt{3} \\ 25/\sqrt{3} \end{bmatrix}$$

or

$$\mathbf{t}^{(\hat{\mathbf{n}})} = \frac{\left(75\hat{\mathbf{e}}_1^* + 50\hat{\mathbf{e}}_2^* + 25\hat{\mathbf{e}}_3^*\right)}{\sqrt{3}}$$

so that

$$\sigma_N = \mathbf{t}^{(\hat{\mathbf{n}})} \cdot \hat{\mathbf{n}} = \frac{1}{3}(75 + 50 + 25) = 50 \text{ MPa}$$

Also, from Eq 3.7-2,

$$\sigma_{\text{oct}}^2 = \mathbf{t}^{(\hat{\mathbf{n}})} \cdot \mathbf{t}^{(\hat{\mathbf{n}})} - \sigma_N^2 = \frac{1}{3}\left[(75)^2 + (50)^2 + (25)^2\right] - (50)^2 = 417$$

and so σ_{oct} = 20.41 MPa. By Eq 3.11-4, we verify directly that

$$\sigma_{\text{oct}} = \frac{1}{3}\sqrt{(75-50)^2 + (50-25)^2 + (25-75)^2} = 20.41 \text{ MPa}$$

Problems

3.1 At a point P, the stress tensor relative to axes $Px_1x_2x_3$ has components σ_{ij}. On the area element $dS^{(1)}$ having the unit normal $\hat{\mathbf{n}}_1$, the stress vector is $\mathbf{t}^{(\hat{\mathbf{n}}_1)}$, and on area element $dS^{(2)}$ with normal $\hat{\mathbf{n}}_2$ the stress vector is $\mathbf{t}^{(\hat{\mathbf{n}}_2)}$. Show that the component of $\mathbf{t}^{(\hat{\mathbf{n}}_1)}$ in the direction of $\hat{\mathbf{n}}_2$ is equal to the component of $\mathbf{t}^{(\hat{\mathbf{n}}_2)}$ in the direction of $\hat{\mathbf{n}}_1$.