3.2 Verify the result established in Problem 3.1 for the area elements having normals

$$\hat{\mathbf{n}}_1 = \frac{1}{7}\left(2\hat{\mathbf{e}}_1 + 3\hat{\mathbf{e}}_2 + 6\hat{\mathbf{e}}_3\right)$$

$$\hat{\mathbf{n}}_2 = \frac{1}{7}\left(3\hat{\mathbf{e}}_1 - 6\hat{\mathbf{e}}_2 + 2\hat{\mathbf{e}}_3\right)$$

if the stress matrix at P is given with respect to axes $Px_1x_2x_3$ by

$$\left[\sigma_{ij}\right] = \begin{bmatrix} 35 & 0 & 21 \\ 0 & 49 & 0 \\ 21 & 0 & 14 \end{bmatrix}$$

3.3 The stress tensor at P relative to axes $Px_1x_2x_3$ has components in MPa given by the matrix representation

$$\left[\sigma_{ij}\right] = \begin{bmatrix} \sigma_{11} & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

where σ_{11} is unspecified. Determine a direction $\hat{\mathbf{n}}$ at P for which the plane perpendicular to $\hat{\mathbf{n}}$ will be stress-free, that is, for which $\mathbf{t}^{(\hat{\mathbf{n}})} = 0$ on that plane. What is the required value of σ_{11} for this condition?

Answer: $\hat{\mathbf{n}} = \frac{1}{3}\left(2\hat{\mathbf{e}}_1 - \hat{\mathbf{e}}_2 - 2\hat{\mathbf{e}}_3\right)$, $\sigma_{11} = 2$ MPa

3.4 The stress tensor has components at point P in ksi units as specified by the matrix

$$\left[\sigma_{ij}\right] = \begin{bmatrix} -9 & 3 & -6 \\ 3 & 6 & 9 \\ -6 & 9 & -6 \end{bmatrix}$$

Determine:

(a) the stress vector on the plane at P whose normal vector is

$$\hat{\mathbf{n}} = \frac{1}{9}\left(\hat{\mathbf{e}}_1 + 4\hat{\mathbf{e}}_2 + 8\hat{\mathbf{e}}_3\right)$$

(b) the magnitude of this stress vector