Determine

(a) the body force components as functions of the coordinates if the equilibrium equations are to be satisfied everywhere

(b) the stress vector at point $P(1,2,3)$ on the plane whose outward unit normal makes equal angles with the positive coordinate axes.

Answers: (a) $b_1 = \dfrac{-3x_2}{\rho}$, $b_2 = \dfrac{-3x_3}{\rho}$, $b_3 = -\dfrac{x_1}{\rho}$

(b) $\mathbf{t}^{(\hat{\mathbf{n}})} = \dfrac{\left(6\hat{\mathbf{e}}_1 + 19\hat{\mathbf{e}}_2 + 12\hat{\mathbf{e}}_3\right)}{\sqrt{3}}$

3.8 Relative to the Cartesian axes $Ox_1x_2x_3$ a stress field is given by the matrix

$$\left[\sigma_{ij}\right] = \begin{bmatrix} \left(1 - x_1^2\right)x_2 + \frac{2}{3}x_2^3 & -\left(4 - x_2^2\right)x_1 & 0 \\ -\left(4 - x_2^2\right)x_1 & -\frac{1}{3}\left(x_2^3 - 12x_2\right) & 0 \\ 0 & 0 & \left(3 - x_1^2\right)x_2 \end{bmatrix}$$

(a) Show that the equilibrium equations are satisfied everywhere for zero body forces.

(b) Determine the stress vector at the point $P(2,-1,6)$ of the plane whose equation is $3x_1 + 6x_2 + 2x_3 = 12$.

Answer: (b) $\mathbf{t}^{(\hat{\mathbf{n}})} = \frac{1}{7}\left(-29\hat{\mathbf{e}}_1 - 40\hat{\mathbf{e}}_2 + 2\hat{\mathbf{e}}_3\right)$

3.9 The stress components in a circular cylinder of length L and radius r are given by

$$\left[\sigma_{ij}\right] = \begin{bmatrix} Ax_2 + Bx_3 & Cx_3 & -Cx_2 \\ Cx_3 & 0 & 0 \\ -Cx_2 & 0 & 0 \end{bmatrix}$$

(a) Verify that in the absence of body forces the equilibrium equations are satisfied.

(b) Show that the stress vector vanishes at all points on the curved surface of the cylinder.

3.10 Rotated axes $Px_1'x_2'x_3'$ are obtained from axes $Px_1x_2x_3$ by a right-handed rotation about the line PQ that makes equal angles with respect to the $Px_1x_2x_3$ axes. Determine the primed stress components for the stress tensor in (MPa)