3.11 At the point P rotated axes $Px_1'x_2'x_3'$ are related to the axes $Px_1x_2x_3$ by the transformation matrix

$$[a_{ij}] = \frac{1}{3}\begin{bmatrix} a & 1-\sqrt{3} & 1+\sqrt{3} \\ 1+\sqrt{3} & b & 1-\sqrt{3} \\ 1-\sqrt{3} & 1+\sqrt{3} & c \end{bmatrix}$$

where a, b, and c are to be determined. Determine $[\sigma_{ij}']$ if the stress matrix relative to axes $Px_1x_2x_3$ is given in MPa by

$$[\sigma_{ij}] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Answer: $$[\sigma_{ij}'] = \frac{1}{9}\begin{bmatrix} 11+2\sqrt{3} & 5+\sqrt{3} & -1 \\ 5+\sqrt{3} & 5 & 5-\sqrt{3} \\ -1 & 5-\sqrt{3} & 11-2\sqrt{3} \end{bmatrix} \text{MPa}$$

3.12 The stress matrix referred to axes $Px_1x_2x_3$ is given in ksi by

$$[\sigma_{ij}] = \begin{bmatrix} 14 & 0 & 21 \\ 0 & 21 & 0 \\ 21 & 0 & 7 \end{bmatrix}$$

Let rotated axes $Px_1'x_2'x_3'$ be defined with respect to axes $Px_1x_2x_3$ by the table of base vectors

	$\hat{\mathbf{e}}_1$	$\hat{\mathbf{e}}_2$	$\hat{\mathbf{e}}_3$
$\hat{\mathbf{e}}_1'$	2/7	3/7	6/7
$\hat{\mathbf{e}}_2'$	3/7	–6/7	2/7
$\hat{\mathbf{e}}_3'$	6/7	2/7	–3/7

(a) Determine the stress vectors on planes at P perpendicular to the primed axes; determine $\mathbf{t}^{(\hat{\mathbf{e}}_1')}$, $\mathbf{t}^{(\hat{\mathbf{e}}_2')}$, and $\mathbf{t}^{(\hat{\mathbf{e}}_3')}$ in terms of base vectors $\hat{\mathbf{e}}_1$, $\hat{\mathbf{e}}_2$, and $\hat{\mathbf{e}}_3$.