(b) Project each of the stress vectors obtained in (a) onto the primed axes to determine the nine components of $\left[\sigma'_{ij}\right]$.

(c) Verify the result obtained in (b) by a direct application of Eq 3.5-1 of the text.

Answer: $$\left[\sigma'_{ij}\right]=\frac{1}{7}\begin{bmatrix}143 & 36 & 114\\ 36 & 166 & 3\\ 114 & 3 & -15\end{bmatrix}\text{ MPa}$$

3.13 At point *P*, the stress matrix is given in MPa with respect to axes $Px_1x_2x_3$ by

$$\text{Case 1: }\left[\sigma_{ij}\right]=\begin{bmatrix}6 & 4 & 0\\ 4 & 6 & 0\\ 0 & 0 & -2\end{bmatrix}\quad \text{Case 2: }\left[\sigma_{ij}\right]=\begin{bmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2\end{bmatrix}$$

Determine for each case

(a) the principal stress values

(b) the principal stress directions.

Answer: (a) Case 1: $\sigma_{(1)}$ = 10 MPa, $\sigma_{(2)}$ = 2 MPa, $\sigma_{(3)}$ = –2 MPa

Case 2: $\sigma_{(1)}$ = 4 MPa, $\sigma_{(2)}$ = $\sigma_{(3)}$ = 1 MPa

(b) Case 1: $$\hat{\mathbf{n}}^{(1)}=\pm\frac{\hat{\mathbf{e}}_1+\hat{\mathbf{e}}_2}{\sqrt{2}},\ \hat{\mathbf{n}}^{(2)}=\pm\frac{\hat{\mathbf{e}}_1-\hat{\mathbf{e}}_2}{\sqrt{2}},\ \hat{\mathbf{n}}^{(3)}=\mp\hat{\mathbf{e}}_3$$

Case 2: $$\hat{\mathbf{n}}^{(1)}=\frac{\hat{\mathbf{e}}_1+\hat{\mathbf{e}}_2+\hat{\mathbf{e}}_3}{\sqrt{3}},\ \hat{\mathbf{n}}^{(2)}=\frac{-\hat{\mathbf{e}}_1+\hat{\mathbf{e}}_2}{\sqrt{2}},\ \hat{\mathbf{n}}^{(3)}=\frac{-\hat{\mathbf{e}}_1-\hat{\mathbf{e}}_2+2\hat{\mathbf{e}}_3}{\sqrt{6}}$$

3.14 When referred to principal axes at *P*, the stress matrix in ksi units is

$$\left[\sigma^*_{ij}\right]=\begin{bmatrix}2 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 12\end{bmatrix}$$

If the transformation matrix between the principal axes and axes $Px_1x_2x_3$ is

$$\left[a_{ij}\right]=\frac{1}{\sqrt{2}}\begin{bmatrix}-\frac{3}{5} & 1 & -\frac{4}{5}\\ a_{21} & a_{22} & a_{23}\\ -\frac{3}{5} & -1 & -\frac{4}{5}\end{bmatrix}$$

where a_{21}, a_{22}, and a_{23} are to be determined, calculate $\left[\sigma_{ij}\right]$.