Answer: $\left[\sigma_{ij}\right] = \begin{bmatrix} 7 & 3 & 0 \\ 3 & 7 & 4 \\ 0 & 4 & 7 \end{bmatrix}$ ksi

3.15 The stress matrix in MPa when referred to axes $Px_1x_2x_3$ is

$$\left[\sigma_{ij}\right] = \begin{bmatrix} 3 & -10 & 0 \\ -10 & 0 & 30 \\ 0 & 30 & -27 \end{bmatrix}$$

Determine

(a) the principal stresses, σ_{I}, σ_{II}, σ_{III}

(b) the principal stress directions.

Answers: (a) $\sigma_{\mathrm{I}} = 23$ MPa, $\sigma_{\mathrm{II}} = 0$ MPa, $\sigma_{\mathrm{III}} = -47$ MPa

(b) $\hat{\mathbf{n}}^{(1)} = -0.394\hat{\mathbf{e}}_1 + 0.788\hat{\mathbf{e}}_2 + 0.473\hat{\mathbf{e}}_3$

$\hat{\mathbf{n}}^{(2)} = 0.913\hat{\mathbf{e}}_1 + 0.274\hat{\mathbf{e}}_2 + 0.304\hat{\mathbf{e}}_3$

$\hat{\mathbf{n}}^{(3)} = 0.110\hat{\mathbf{e}}_2 + 0.551\hat{\mathbf{e}}_2 - 0.827\hat{\mathbf{e}}_3$

3.16 At point *P*, the stress matrix relative to axes $Px_1x_2x_3$ is given in MPa by

$$\left[\sigma_{ij}\right] = \begin{bmatrix} 5 & a & -a \\ a & 0 & b \\ -a & b & 0 \end{bmatrix}$$

where *a* and *b* are unspecified. At the same point relative to axes $Px_1^*x_2^*x_3^*$ the matrix is

$$\left[\sigma_{ij}^*\right] = \begin{bmatrix} \sigma_{\mathrm{I}} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \sigma_{\mathrm{III}} \end{bmatrix}$$

If the magnitude of the maximum shear stress at *P* is 5.5 MPa, determine σ_{I} and σ_{III}.

Answer: $\sigma_{\mathrm{I}} = 7$ MPa, $\sigma_{\mathrm{III}} = -4$ MPa

3.17 The state of stress at point *P* is given in ksi with respect to axes $Px_1x_2x_3$ by the matrix

$$\left[\sigma_{ij}\right] = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & -2 \end{bmatrix}$$