Determine

(a) the principal stress values and principal stress directions at P

(b) the maximum shear stress value at P

(c) the normal $\hat{\mathbf{n}} = n_i \hat{\mathbf{e}}_i$ to the plane at P on which the maximum shear stress acts.

Answer: (a) $\sigma_{(1)} = 2$ ksi, $\sigma_{(2)} = 1$ ksi, $\sigma_{(3)} = -3$ ksi

$$\hat{\mathbf{n}}^{(1)} = \frac{2\hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_3}{\sqrt{5}}, \quad \hat{\mathbf{n}}^{(2)} = \hat{\mathbf{e}}_2, \quad \hat{\mathbf{n}}^{(3)} = \frac{-\hat{\mathbf{e}}_1 + 2\hat{\mathbf{e}}_3}{\sqrt{5}}$$

(b) $(\sigma_S)_{\max} = \pm\ 2.5$ ksi

(c) $\hat{\mathbf{n}} = \dfrac{\hat{\mathbf{e}}_1 + 3\hat{\mathbf{e}}_3}{\sqrt{10}}$

3.18 The stress tensor at P is given with respect to $Ox_1x_2x_3$ in matrix form with units of MPa by

$$[\sigma_{ij}] = \begin{bmatrix} 4 & b & b \\ b & 7 & 2 \\ b & 2 & 4 \end{bmatrix}$$

where b is unspecified. If $\sigma_{\mathrm{III}} = 3$ MPa and $\sigma_{\mathrm{I}} = 2\sigma_{\mathrm{II}}$, determine

(a) the principal stress values

(b) the value of b

(c) the principal stress direction of σ_{II}.

Answer: (a) $\sigma_{\mathrm{I}} = 8$ MPa, $\sigma_{\mathrm{II}} = 4$ MPa, $\sigma_{\mathrm{III}} = 3$ MPa

(b) $b = 0$, (c) $\hat{\mathbf{n}}^{(\mathrm{II})} = \hat{\mathbf{e}}_1$

3.19 The state of stress at P, when referred to axes $Px_1x_2x_3$ is given in ksi units by the matrix

$$[\sigma_{ij}] = \begin{bmatrix} 9 & 3 & 0 \\ 3 & 9 & 0 \\ 0 & 0 & 18 \end{bmatrix}$$

Determine

(a) the principal stress values at P

(b) the unit normal $\hat{\mathbf{n}}^* = n_i \hat{\mathbf{e}}_i^*$ of the plane on which $\sigma_N = 12$ ksi and $\sigma_S = 3$ ksi

Answers: (a) $\sigma_{\mathrm{I}} = 18$ ksi, $\sigma_{\mathrm{II}} = 12$ ksi, $\sigma_{\mathrm{III}} = 6$ ksi

(b) $\hat{\mathbf{n}}^* = \dfrac{\hat{\mathbf{e}}_1^* + \sqrt{6}\hat{\mathbf{e}}_2^* + \hat{\mathbf{e}}_3^*}{2\sqrt{2}}$