Security-Constrained Optimal Power and Natural-Gas Flow

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***Abstract*—Continuous liberalization and interconnection of energy markets worldwide has raised concerns about the inherent interdependency between primary energy supply and electric systems. With the growing interaction among energy carriers, limitations on the fuel delivery are becoming increasingly relevant to the operation of power systems. This paper contributes with a novel formulation of a mixed-integer linear programing (MILP) security-constrained optimal power and gas flow. To this end, an iterative methodology, based on development of linear sensitivity factors, determines the stabilized post-contingency condition of the integrated network. The proposed model allows system operators not only to perform security analysis but also to adjust in advance state variables of the integrated system optimally and fast, so that $n-1$ contingencies do not result in violations. Case studies integrate the IEEE 24-bus system and a modified Belgian high-calorific gas network for analyzing the performance of the formulation and solution methodology.**

***Index Terms*—Integrated energy systems, natural gas networks, optimal power flow, security analysis, security constrained.**

NOMENCLATURE

A. Sets and Indices

i, j, i', j'	Gas nodes.
k	Piecewise segments for gas flows.
l	Transmission lines.
m, n, m', n'	Power nodes.
$p, p'/pa$	Passive/active pipelines.
s	Storage facilities.
u	Generators.
w	Gas wells.

B. Parameters

$B_{p,k}$	Intercept of piecewise linear segment [Sm^3/d].
C_p	Weymouth constant.
CG_w, CS_s	Cost of gas production and storage [\$/Sm^3].
CP_u	Cost of power generation [\$/MWh].
E_p	Intercept of equation for sensitivities [Sm^3/d].
EF_u	Efficiency factor [Sm^3/MWh].
$\overline{FP}_l$	Line capacity [MW].
$\overline{FL}_{p,k}, \underline{FL}_{p,k}$	Max/Min gas flow of linear segments [Sm^3/d].
GL_i, PL_m	Gas and power demand [Sm^3/d]/[MW].
H_p	Slope of linear gas flow equation for sensitivities.
IR_s, OR_s	Max. in- and out-flow rates [Sm^3/d].
$M_{p,k}$	Slope of piecewise linear segment.
PD	Penalty for overpressure in post-contingency pressure squared [\$/bar^2].
PG	Penalty for non-served gas [\$/Sm^3].
PP	Penalty for non-served power [\$/MWh].
$\overline{S}_s, \underline{S}_S$	Max/Min storage level [Sm^3].
$\overline{U}_u, \underline{U}_u$	Max/Min power generation [MW].
V	Matrix used to compute NG factors [pu].
$\overline{W}_w, \underline{W}_w$	Max/Min gas production [Sm^3/d].
X_l	Line impedance [pu].
Z	Inverse of the admittance matrix [pu].
$\overline{\pi}_i, \underline{\pi}_i$	Max. and Min. pressure squared [bar^2].
Γ_p	Compression factor [bar^2].

C. Variables

$d^{ng}_{p,p'}, d^{pw}_{l,l'}$	Pipeline/line outage distribution factors.
fg_p	Pre-contingency gas flow [Sm^3/d].
$fl_{p,k}$	Piecewise linear gas flow [Sm^3/d].
fp_l	Pre-contingency power flow [MW].
$\widetilde{fg}_p$	Post-contingency gas flow [Sm^3/d].
nd_i	Non-delivered gas [Sm^3].
np_m	Non-served energy [MWh].
$o_{p,k}$	Status of piecewise linear segment $(binary)$.
ps_i	Pressure [bar].
pg_w	Gas production [Sm^3/d].
pw_u	Power generation [MW].
sv_s	Storage volume [Sm^3].

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sf_s	Storage flow [Sm^3/d].
$\delta^{ng}_{i',p}$	Pressure squared shift factors.
π_i	Pre-contingency pressure squared [bar^2].
$\tilde{\pi}_i$	Post-contingency pressure squared [bar^2].
θ_m	Node angle [rad].
$\triangle\tilde{\pi}_{i,p}, \nabla\tilde{\pi}_{i,p}$	Positive/negative overpressure squared in post-contingency [bar^2].

I. Introduction

A. Motivation

THE integration of energy sources in planning and operational procedures is motivated by the evolution and change that energy systems have experimented around the world. Integrated energy systems (IES) [1] are currently affected not only by efficiency reasons and better risk allocation arising from deregulation, but also by higher reliability challenges. Analysis of IES reliability has become more relevant since interdependency between systems is increasing every day and larger energy networks are being developed, as shown in [2], [3] for U.S. and in [4] for Trans-European networks.

A particular case of IES is the integration of power and natural gas (NG) networks. Several authors [2], [5]–[8] agree that interdependence between these systems has increased recently because of the significant growth in gas-fired units, especially combined-cycle plants. Accordingly, the power system security is been impacted in such a way that gas outages or gas supply limitations may force multiple units to go offline [7] and reliability conditions may be compromised for both systems when demands peak simultaneously [2]. Recent events such as those in Colorado 2006,[1] Texas 2011[2] or Colombia 2012[3] are real examples that evidence current security issues. This situation could become more critical since by 2030 gas-fired generation is expected to increase by 230% [9].

As a result, contingency-analysis programs and security-constrained models are required to operate both power and NG systems economically and securely so that single contingencies in the IES do not result in violations. This conclusion agrees with the 2006 reliability study performed by NERC [5], which emphasizes the importance of NG delivery to power system reliability. Also, this corporation and FERC commissioner Moeller in 2013 [10] call for increased study of the reliability and adequacy of systems as a result of unexpected fuel transportation contingencies and the growing physical and functional ties between electric and NG systems.

B. Literature Review

Several papers have demonstrated the necessity to integrate power and NG networks [8], [5], [11], [12] and others have modeled the NG network with flows and pressures using either nonlinear [13]–[15] or linearized formulations [5], [16], [17].

Specifically regarding to the IES security analysis, only the power system security has been evaluated. [6] focuses on the short-term operation of gas/electric composite system and evaluate, among others, the consequences of gas system failures on the electricity market operation; but the NG network is not modeled. Likewise, [2] and [18] incorporate NG network constraints into the solution of the security-constrained unit commitment. The first tests the feasibility of different solutions in a nonlinear NG network by applying the Newton-Raphson method; the second presents a stochastic model with random outages of system components for the coordination of midterm water and natural gas supplies; and the last uses a simplified NG network model based on consumption and production zones. The only contribution addressing contingencies in the IES to evaluate the behavior of the integrated system is presented in [19]. This reference develops a stochastic optimization to analyze the effects of network uncertainties in the short-term operation of the integrated system. However, additional efforts are required to study the security of NG networks as it is for power systems in a way that operators are prepared to address further challenges.

[1]"2006 System Disturbances", published by NERC 2/20/07

[2]http://ourfiniteworld.com/2011/02/04/texas-electricity-trying-to-understand-the-blackouts/

[3]http://www.dinero.com/actualidad/pais/articulo/en-costa-caribe-miles-usuarios-tienen-racionamiento-gas-luz-agua/143028

C. Contributions and Paper Organization

The main contributions of this paper are:

1) The formulation of a MILP for the security-constrained optimal power and NG flow. This model combines a contingency-analysis of the integrated network with an optimal power and NG flow. Consequently, preventive adjustments to the optimal generation and gas production, among others, are carried out to minimize overloads when contingencies occur.
2) The development of a contingency-analysis for the NG system using linear sensitivity factors. This approximation allows to provide a quick calculation of possible network violations when $n-1$ contingencies are studied.
3) Case studies illustrate the solution of the proposed formulation over a range of different operating conditions.

Moreover, this proposal is expected to be used as the fundamental basis for future multi-period research such as unit commitment formulations, in which the dynamics of NG and line-packing capabilities are considered.

The remainder structure of this paper is as follows: Section II develops the basic theory about security analysis for NG systems. Section III describes the problem formulation. Section IV proposes a methodology to calculate linear sensitivity factors for outages in NG systems and to solve the integrated optimization model. Section V shows results and analysis of case studies. Finally, Section VI draws main conclusions.

II. Security Analysis of NG Systems

A. Key Components of NG Infrastructure

This section presents a brief description of each NG network component used in this paper. The interested reader is referred to [2], [6] and [20] for more detailed information. Fig. 1 depicts the basic NG network components from production to consumption. The NG network modeling developed further on contains decision and state variables of production, flows, pressures and storage. Similarly to power systems, flows are associated with branches and pressures with nodes.

Gas wells (production): Most of the gas is supplied from wells or regasification terminals located remotely.

Consumers (loads): They can be classified into electrical (gas-fired units) and non-electrical (remainders) costumers.

Compressors: Devices to maintain desired pressure levels in pipelines. Compressor as a branch in the NG transmission

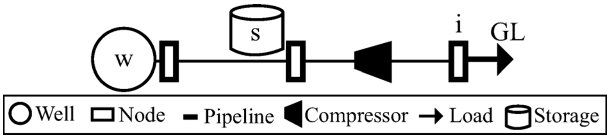

Fig. 1. Basic NG network components.

system is analogous to a phase shifter or transformer in power networks [6].

Pipelines: They transport NG from producers to consumers. Contrary to power systems where all transmission lines have the same modeling, NG pipelines are classified as passive (simple pipelines) or active (pipelines with a compressor).

Storages: Unlike power systems in which the electricity cannot be stored in significant quantities, large sums of NG can be injected into these facilities in order to serve peak demand and to maintain a steady flow through pipelines during contingencies [6].

B. Security Analysis

Following security analysis for NG systems is developed similarly to the one discussed in [21] for power networks. Studies of the system security aims to develop practices in order to keep the system operating when components fail. Taking into account that outages are unpredictable, the system must be operated at all times in a way that security levels remain even after any failure, thus preventing cascading events. Hence, any single contingency in the NG network should not leave any component heavily overloaded accomplishing the $n-1$ security criteria.

Large NG networks composed of thousands of elements are more likely to suffer contingencies. Network disruptions can be caused by both natural and human factors, such as: weather conditions (e.g., extreme frosts or rains, earthquakes, etc.), pipelines ruptures (e.g., due to land excavations, vandalism, etc.) or bad operations during maintenance activities. Accordingly, some failures may result in either elements operating beyond their limits or gas supply curtailments.

There are three major functions carried out by system operators in order to operate the system safely [21]: System monitoring, contingency analysis and security-constrained optimal energy flow (SCOEF). The first function is beyond the scope of this paper but the other two are discussed below.

Contingency analysis allows the system to be operated defensively. In order to prevent cascading failures, contingency analysis programs seek to detect possible problems in the network before they arise. Hence, credible outages are simulated and any potential pressure out-of-limit or overflow must be detected. The problem of studying $n-1$ credible failures becomes very difficult to solve if it is desired to analyze results quickly. Consequently, a methodology based on linear sensitivity factors is used to represent the sensitivity of any state variable (such as pressures or flows) to a change in another variable (pressures, flows or production) in a simple and fast way. These factors are not exact since they require a linearization of the relationship between flows and pressures and can only be applied to meshed networks assuming that flow and pressure variables change instantaneously. In reality, when a fault occurs the pressure waves travel at the speed of sound in gas but the gas travel velocity depends on the flow and the pipelines' characteristics [20]. As a consequence, the proposed methodology aims to estimate conservative values for an optimal energy flow in order to provide safe operations when the system is stabilized in the post-contingency state.

Lastly, security-constrained optimal gas flows combine the contingency analysis program with an optimal gas flow seeking to adjust economically production, pressure levels, storage and compressor operations, thus single contingencies do not result in violations. In particular for NG systems, security in nodes must be carefully monitored as pressures determine gas flows through pipelines [20] and quantities to be delivered.

III. Problem Formulation

The security-constrained optimal power and gas flow is formulated as follows: First, the optimal power and gas flow model is presented, where the electric and NG transmission systems are coupled through gas-fired generators. Afterwards, the linear sensitivity factors are developed for the NG systems and finally, contingency-constraints are introduced to prevent overloaded elements in the post-contingency state.

A. Optimal Power and Gas Flow (OEF)

This model seeks the minimum operation cost for power generation and gas production while energy flows are balanced through transportation networks.

1) Objective Function: It minimizes the sum of operational costs and the penalties associated with non-served gas and power and post-contingency overpressure squared in the node i when pipeline p fails:

$$\min \sum_w CG_w \cdot pg_w + \sum_s CS_s \cdot sv_s + \sum_u CP_u \cdot pw_u + \sum_i PG \cdot nd_i + \sum_m PP \cdot np_m + \sum_{p,i} PD \cdot (\triangle\tilde{\pi}_{i,p} + \nabla\tilde{\pi}_{i,p}). \quad (1)$$

2) Operating Constraints: Power system: The generation of each unit is limited by maximum and minimum parameters:

$$\underline{U}_u \leq pw_u \leq \overline{U}_u \quad \forall u. \quad (2)$$

NG Production and consumption capacity: Similarly to power systems, gas production is bounded by either physical characteristics or contracted amounts:

$$\underline{W}_w \leq pg_w \leq \overline{W}_w \quad \forall w. \quad (3)$$

NG storage: Inventory constraints and maximum levels for storages are represented in (4). Positive values of storage flows (sf) correspond to outflows and negative values to inflows. Although maximum in- and out-flow rates of the storage vary with the current level [20], this paper considers a simplified formulation with single in-flow and outflow rates (5):

$$\underline{S}_s \leq sv_s = sv_s^0 - sf_s \leq \overline{S}_s \quad \forall s \quad (4)$$

$$-IR_s \leq sf_s \leq OR_s \quad \forall s. \quad (5)$$

3) Network Constraints: Nodal balance: Both power (6) and natural gas (7) networks must accomplish the nodal supply-demand balance. Note that (7) couples NG and power networks by including the gas consumed by each power unit according to its efficiency factor. The interested reader is referred to [22] for a more detailed formulation regarding the heat rate curve:

$$\sum_l fp_l + \sum_u pw_u + np_m = PL_m \quad \forall m \quad (6)$$

$$\sum_{p} fg_p + \sum_{w} pg_w + \sum_{s} sf_s + nd_i - pw_u \cdot EF_u = GL_i \quad \forall i, u \in i. \quad (7)$$

Power system network constraints: The flow in the DC formulation is given by

$$-\overline{FP}_l \leq fp_l = \frac{1}{X_l}(\theta_m - \theta_n) \leq \overline{FP}_l \quad \forall l, (m,n) \in l. \quad (8)$$

NG Flow and Pressure constraints: The modeling of pressure drops in pipelines is based on the nonlinear Weymouth (9) [20]. Gas flows are unrestricted in sign and the constant C_p depends on the gas composition and length, diameter and absolute rugosity of pipeline (further details can be found in [17] and [20]):

$$\operatorname{sign}(fg_p) \cdot fg_p^2 = C_p^2 \left(ps_i^2 - ps_j^2\right). \quad (9)$$

However, this paper uses a linear approximation in order to harness the computational advantages of linearity. Since the pressure appears in the problem only with a power of 2 in (9), a simple substitution in which $\pi = ps^2$ eliminates this nonlinearity. The left side of (9) can be represented as a piecewise linear function [5]. Each function is modeled by a slope M, an axis intercept B and a binary variable o representing its segment status. As a result, (9) is formulated linearly through (10)–(13):

$$\sum_k (M_{p,k} \cdot fl_{p,k} + B_{p,k} \cdot o_{p,k}) = C_p^2(\pi_i - \pi_j) \quad \forall p, (i,j) \in p \quad (10)$$

$$o_{p,k} \cdot \underline{FL}_{p,k} \leq fl_{p,k} \leq o_{p,k} \cdot \overline{FL}_{p,k} \quad \forall k, p \quad (11)$$

$$fg_p = \sum_k fl_{p,k} \quad \forall p \quad (12)$$

$$\sum_k o_{p,k} \leq 1 \quad \forall p. \quad (13)$$

In order to formulate an accurate and secure model, the points comprising the linear segments should be placed over the Weymouth equation. Finally, quadratic pressure levels in each node are bounded by

$$\underline{\pi}_i \leq \pi_i \leq \overline{\pi}_i \quad \forall i. \quad (14)$$

NG compressors: For active pipelines, the pressure at the incoming node i is lower than the pressure at the out-coming node j and the gas flows from i to j. Therefore, (10) is relaxed, the gas is forced to flow in one direction ($fg_p \geq 0$) and the pressure at the exit of each compressor is bounded according with the compression factor (15):

$$\pi_j \leq \Gamma_p \cdot \pi_i \quad \forall p \in pa, (i,j) \in p. \quad (15)$$

B. Contingency Constraints

The following equations are integrated to the OEF in order to bound post-contingency conditions according with the procedure presented in Section IV. Linear sensitivity factors and post-contingency variables are derived in Appendix A. Constraints (16) are defined for gas flows and (17) for pressure squared in which overpressure, when pipeline p' is failed, are allowed to avoid additional non-served gas when contingencies occur. Similarly, (18) bounds post-contingency power flows through line l to a contingency of line l':

$$\underline{FL}_{p,k=1} \leq fg_p + d_{p,p'}^{ng} \cdot fg_{p'} \leq \overline{FL}_{p,k=K} \quad (16)$$

$$\underline{\pi}_i \leq \pi_i + \delta_{i,p'}^{ng} \cdot fg_{p'} - \triangle\tilde{\pi}_{i,p} + \nabla\tilde{\pi}_{i,p} \leq \overline{\pi}_i \quad (17)$$

$$-\overline{FP}_l \leq fp_l + d_{l,l'}^{pw} \cdot fp_{l'} \leq \overline{FP}_l. \quad (18)$$

IV. Solution Methodology

A. Estimating the Post-Contingency State

Linear sensitivity factors are formulated to be able to include the $n-1$ criteria and obtains results quickly. This methodology also allows system operators to detect network problems before a contingency occurs and adequate the system preventively. In order to derive NG sensitivity factors, similarly as [21] does it for the power system, the pressures are assumed to change instantaneously, radial circuits must be excluded from the set of credible contingencies and the Weymouth equation is linearized (19)

$$fg_p = E_p + H_p(\pi_i - \pi_j). \quad (19)$$

Taking into account that post-contingency flows and pressures derived from (19) must also fulfill the nodal balance equation, E and H parameters are calculated from previous balanced pre-contingency values obtained from the OEF. Otherwise, neither the gas nodal balance nor (19) will be satisfied in post-contingency. As a consequence, the NG security-constrained problem becomes iterative if linearity is desired to be preserved. Particularly, this situation does not exist in the power system because its sensitivity factors are estimated from a constant relationship given by the impedance parameter.

Firstly, in order to understand how the linear sensitivities work, Fig. 2 depicts, for a given pipeline, the evolution of post-contingency variables from the pre-contingency operating point. In addition, pressure-squared differences for an incremental post-contingency flow are represented when different approximations are used to estimate their value. The points marked in the figure correspond to:

1) The pre-contingency operating point obtained from the OEF. From this condition, the post-contingency flow increases by an amount given by the flow through the dropped pipeline and the sensitivity factor (16).
2) The operating point if the post-contingency state is modeled by the Weymouth equation (accurate value). In this state, the post-contingency pressure-squared difference $d\tilde{\pi}_r$ is obtained.
3) The operating point when the linear (19) approximates the post-contingency state. Here, the post-contingency pressure-squared difference $d\tilde{\pi}_s$ results.

Notice that all post-contingency operating points placed in the gray zone can be considered as conservative since the linear model will always find, for a given flow, greater pressure-squared differences than the real ones ($d\tilde{\pi}_s > d\tilde{\pi}_r$). Greater pressure-squared differences are relevant for security analysis because they mean that the pipeline is being operated close to its limit. Consequently, in order to estimate more accurate and secure pipelines operations, different linear equations are formulated according to the direction followed by the post-contingency flow, as shown in Fig. 3. This figure depicts the secure linear equations that must be used to model possible

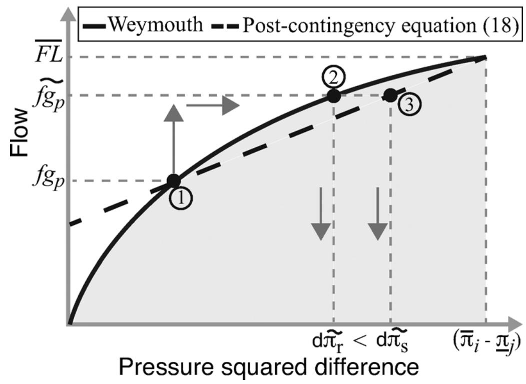

Fig. 2. Post-contingency variables for an increasing flow.

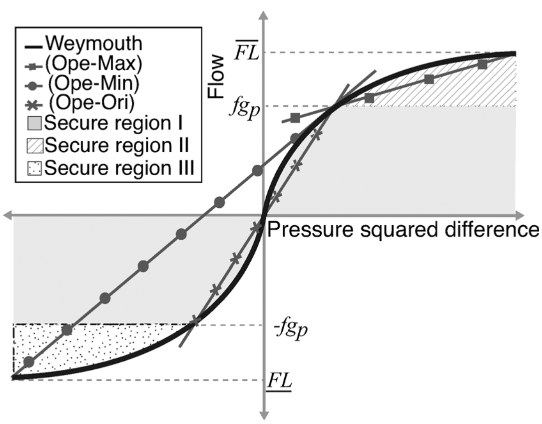

Fig. 3. Approximations for different post-contingency flows.

post-contingency flows derived from a positive pre-contingency flow. A similar analysis can be done for post-contingency flows arising from negative pre-contingency flows. According with the region where post-contingency flows are placed, the approximations presented in Table I are used to calculate the line parameters.

When studying the dynamics of NG systems associated with the gas travel velocity, multi-period models should be formulated in which time-delay constants are incorporated into the calculation of the flows resulting from the methodology presented in this paper.

B. Implementation of the Security-Constrained Optimal Power and Gas Flow

Considering that E and H parameters are function of pre-contingency variables, an iterative algorithm is presented in Fig. 4 in order to incorporate progressively active contingency-constraints in the OEF while the model linearity is preserved. The main processes of the flowchart are:

1) The OEF [(1)–(8) and (10)(14)–(15)] is solved with active contingency-constraints [(16)–(18)].
2) Sensitivity factors and post-contingency variables are calculated according with pre-contingency results. For the NG system, E and H are adjusted by the best linear equation that considers, for each failure, the evolution of post-contingency flows and the secure region where they are placed. For the power system, the formulation presented in Appendix A to compute (18) is included.
3) If any of the post-contingency variables is beyond its limits, a new active contingency-constraint [(16), (17) and/or (18)] is added to the OEF thus limiting the pre-contingency variables.
4) This loop is executed until no new violations appear or a maximum number of iterations is reached. Then, system operators are alarmed about specific contingencies and variables out of bounds that could compromise the network security.

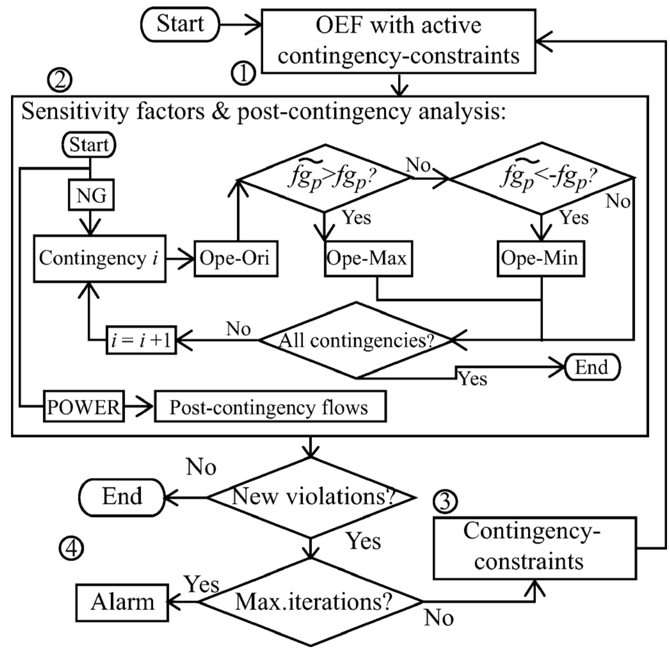

Fig. 4. Security-constrained optimal power and gas flow.

It is relevant to note that NG contingency-constraints must be added cumulatively in order to avoid infinite loops in the algorithm. Moreover, when a set of possible contingencies is taken into account, post-contingency flows for a certain pipeline can behave different for each situation: for some contingencies the flow can increase but for others can decrease, and even can change its sign. This phenomenon will be identified as "varying flows". As a result, pipelines are approximated with the best linear equation (Ope-Ori, Ope-Max or Ope-Min) for each contingency. This means that parameters E and H are calculated for each failure included in the $n-1$ analysis. An exception is considered for active pipelines. They are always modeled by Ope-Max because their flow is always positive and a value of E different than zero is required when negative pressure-squared differences result.

Although the equations presented in Table I are formulated in a general way by considering the technical parameters as maximum and minimum flow and pressure-squared differences, these values can be adjusted to improve the model accuracy. Consequently, maximum and minimum post-contingency flows used to approximate the post-contingency state of pipeline p when p' fails, are redefined by (20) and (21). Then, pressures squared are calculated from (9):

$$\overline{FL}_{p,k=K} = \min(\overline{FL}_{p,k=K}, fg_p + fg_{p'}) \tag{20}$$
$$\underline{FL}_{p,k=1} = \max(\underline{FL}_{p,k=1}, fg_p - fg_{p'}). \tag{21}$$

V. CASE STUDIES

Two case studies were applied in order to illustrate the functioning of the proposed optimization model and the solution

TABLE I
PARAMETERS TO CALCULATE SECURE NG SENSITIVITY FACTORS

Region	Line	H_p	E_p
I	Ope-Ori	$\frac{fp_p}{(\pi_i-\pi_j)}$	0
II	Ope-Max	$\frac{\overline{FL}_{p,k=K}-fp_p}{(\overline{\pi}_i-\underline{\pi}_j)-(\pi_i-\pi_j)}$	$fp_p - H_p(\pi_i - \pi_j)$
III	Ope-Min	$\frac{\underline{FL}_{p,k=1}-fp_p}{(\underline{\pi}_i-\overline{\pi}_j)-(\pi_i-\pi_j)}$	$fp_p - H_p(\pi_i - \pi_j)$

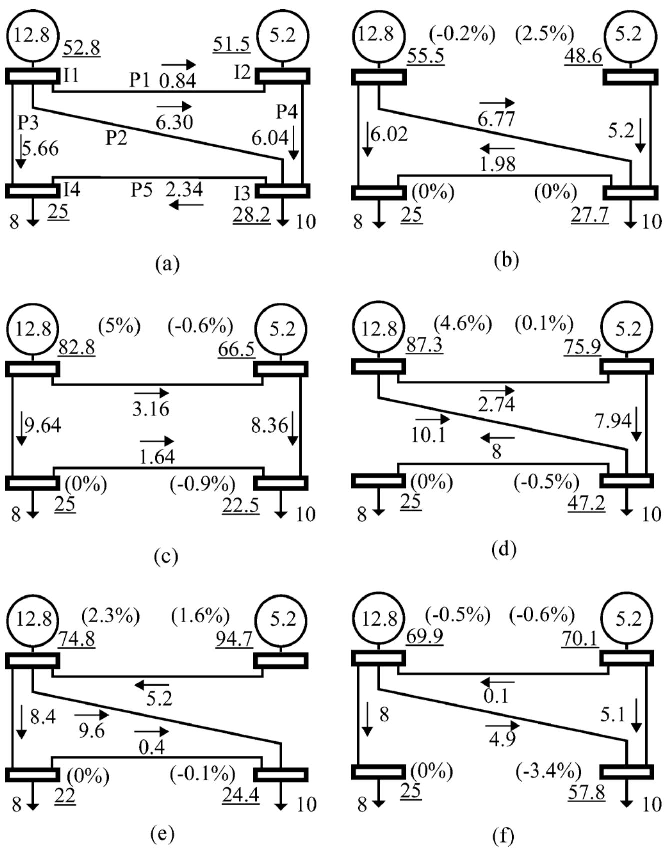

Fig. 5. 4-node NG pre-contingency and post-contingency results. Flows in [Sm^3/d] and pressures in [bar]. (a) Pre-contingency, (b) Post-contingency—P1 failed, (c) Post-contingency—P2 failed, (d) Post-contingency—P3 failed, (e) Post-contingency—P4 failed, (f) Post-contingency—P5 failed.

methodology. All the formulation and algorithms were implemented in GAMS and the optimization was carried out using CPLEX 12.5.

Results obtained from the proposed model are compared with a reference case, which is formulated as an OEF that optimizes unique production variables not only for the steady state but also for all possible post-contingency states. Network basic data can be downloaded from http://db.tt/OvMo70Q9.

A. Four-Node NG System

The $n-1$ criteria is applied to the NG system shown in Fig. 5 to clarify the basic concepts of the methodology proposed. The security-constrained optimal formulation is validated when all pipelines are failed.

Nodal pressures are underlined and their percentage of post-contingency deviations are compared against the reference case. Positive and negative pressure deviations are related to flows differences caused by the linearization introduced to estimate the post-contingency state. However, obtained results are conservative because all pressure-squared differences calculated by the model are greater than those obtained by the reference case. In addition, all production scheduled by the model in pre-contingency is delivered in post-contingency with lower over pressures than the reference simulation. These over pressures are identified by the algorithm and could be avoided by reducing the production and scheduling non-served gas preventively. On the other hand, the percentage of deviation in node pressures demonstrates that the results are accurate. In this case study, all pipelines have varying flows and all linear approximations (Ope-Ori, Ope-Max and Ope-Min) are used to estimate the post-contingency condition.

B. Integrated Energy System

The IES presented in Fig. 6 is composed by the IEEE 24-bus power system from [23] and a modified version, considering [24], of the Belgian high-calorific 20-node gas network detailed in [17]. Credible individual contingencies in pipelines P7, P11, P21 and transmission lines L7 or L20 are assumed to illustrate the proposed model and methodology in larger IES.

Wells and storages injections are underlined in Fig. 6. As a consequence of varying flows that appeared for different failures, all approximations (Ope-Ori, Ope-Max and Ope-Min) are used to estimate the post-contingency scenarios. Figs. 7 and 8 present the deviations in node pressures and flows when the results calculated by the model are compared against the reference simulation. All pressure-squared differences obtained by the model, when contingencies occur, are greater than in the reference. A maximum pressure deviation of 1.7% and a maximum flow deviation of 3.5% illustrate the accuracy of the model. Accordingly, the different contingencies are approximated conservatively, securely and precisely.

Results showing the interaction between energy systems, when the set of credible contingencies is analyzed, are presented in Table II. The production when an OEF is executed with no contingencies is compared with the one calculated by the proposed model (SCOEF). The failure of L20 reduces the generation of U4 to 79.8 MW. Gas-fired units U1 and U5 balance this reduction. Although U5 is cheaper than U1, is not capable of producing all the required power because the outage of L7 limits its production. These changes on the generation imply the following adjustments in the balance of the NG system: the production of W1 is reduced to 13.6 Sm^3 due to the curtailment in U4, and the production in W2 is increased to 18.7 Sm^3 in order to balance the new production in U1 and U5. This NG balance considers that the contingency of P7 limits the production of W1 and outages of P11 and P21 lead to curtailments of W2.

The impact of conservative results are presented in Table III. In comparison with the reference simulation, the proposed model resulted in an increment of 1.2% in the objective function for the IES case study, which has not had pressure squared deviations. Otherwise, a 9% of the 12% increase in the objective function of the 4-node NG case study was due penalizations associated with pressure squared deviations. Lastly, the execution time of the more complex model, the IES, took 23% less time by the developed algorithm. In this case study the optimization is more complex due to the size of the problem. Otherwise for the simple case study, the 4-node NG system, the model took 63% more time than the reference because the iterative process consumed more time than the optimization.

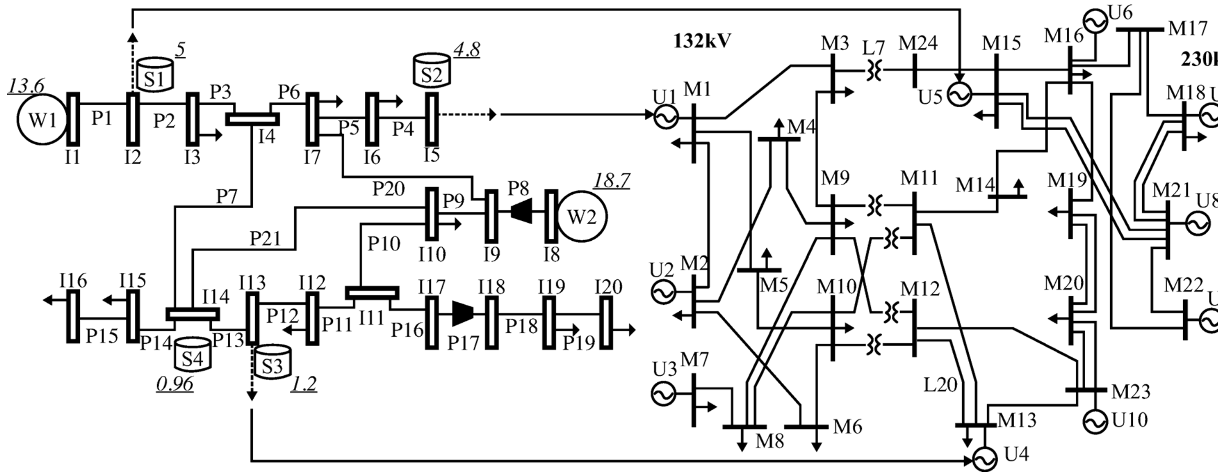

Fig. 6. IES composed by a modified 20-node Belgian NG network and 24-bus IEEE power system. Injections in [Sm^3/d].

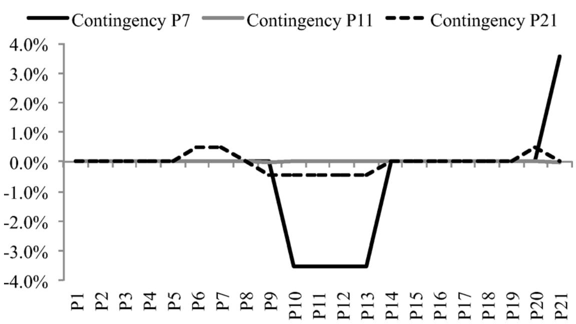

Fig. 7. NG flow deviations for the IES.

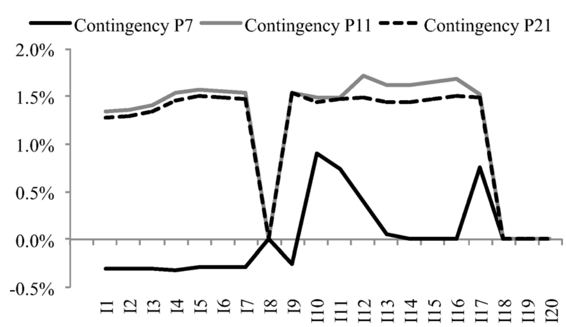

Fig. 8. Pressure deviations for the IES.

TABLE II
PRODUCTION IMPACT: OEF VERSUS SCOEF

	W1	W2	U1	U4	U5
	[Sm^3]		[MW]		
OEF	15.0	17.5	0.0	293.0	0.0
SCOEF	13.6	18.7	192.0	79.8	21.1

TABLE III
RESULTS IMPACT: MODEL VERSUS REFERENCE

	Objective	Pressure deviations	Execution time
4-node NG	12%	9%	-63%
IES	1.2%	0%	23%

VI. CONCLUSION

The main contribution of this paper is the first development of a MILP security-constrained optimal power and gas flow. This formulation allows system operators to adjust optimally and quickly state variables of the integrated system, so that $n-1$ contingencies do not result in violations. To this end, a methodology is proposed to calculate linear sensitivity factors and then NG contingency-constraints to estimate fast and secure post-contingency states for the integrated system. Although this methodology assumes that the system in post-contingency state is stabilized instantaneously, obtained results showed that all post-contingency pressures in such situation are operated precisely, conservatively and securely. Therefore, the quickness and accuracy of this proposal suggests that it can be used as the fundamental basis for further research to study the dynamics of the NG with multi-period formulations, such as unit commitment models applied to IES. Moreover, the developed algorithm can be also used by system operators to perform in advance contingency-analysis simulations, operating the IES defensively and preventing cascading events.

APPENDIX

A. NG Linear Sensitivity Factors

Equation (19) can be written in matrix form as (22). Considering that E_p parameters are known data, the change of E with respect to node gas injections is zero, which results in (23). This equation has the same form to that obtained in [21] for power systems, then the same procedure to derive the different sensitivity factors can by applied for NG systems. In addition, matrixes $[H]$ and $[V]$, which are analogous to matrixes $[B]$ and $[X]$ used in [21] for the power system, are calculated according to (24) and (25):

$$[\pi] = [V]\cdot([fg]-[E]) \tag{22}$$

$$\triangle[\pi] = [V]\cdot\triangle[fg] \tag{23}$$

$$[V] = [H]^{-1} \tag{24}$$

$$[H_{ij}] = -H_{ji}, \quad [H_{ii}] = \sum_{i=1}^{I} H_{ij}. \tag{25}$$

As a consequence of linearity, the effects of simultaneous changes on several state variables can be calculated using superposition and a pipeline outage is modeled by adding two gas injections to the system, one at each end of the pipe to be dropped without removing the element from the network.

Transport Outage Distribution Factors: Similarly to line outage distribution factors (26) derived in [21] for the power system, pipeline outage distribution factors (27) are defined for the NG as the sensitivity of the flow on pipeline p' (from node i' to j') to a change in the flow of pipeline p:

$$d_{l',l}^{pw} = \frac{\triangle fp_{l'}}{fp_l} = \frac{X_l}{X_{l'}}\left[\frac{Z_{n'n} - Z_{n'm} - Z_{m'n} + Z_{m'm}}{X_l - (Z_{nn} + Z_{mm} - 2Z_{nm})}\right] \tag{26}$$

$$d_{p',p}^{ng} = \frac{\triangle fg_{p'}}{fg_p} = \frac{H_{p'}(V_{i'i} - V_{i'j} - V_{j'i} + V_{j'j})}{1 - H_p(V_{ii} + V_{jj} - 2V_{ij})}. \tag{27}$$

Pressure Shift Factors: Considering that pressures are a key safety variable that must be included in the contingency analysis, pressure squared shift factors (28) are developed to represent the sensitivity of the pressure squared in node i' to the original gas flow over a pipeline p before it fails. These factors are similar to those defined in [21] representing changes in phase angles to the power flowing over a line before it is dropped:

$$\delta_{i',p}^{ng} = \frac{\triangle\pi_{i'}}{fg_p} = \frac{V_{i'i} - V_{i'j}}{1 - H_p(V_{ii} + V_{jj} - 2V_{ij})}. \tag{28}$$

B. Post-Contingency Conditions

In parallel to power systems, by using the linear sensitivity factors, post-contingency gas flows and pressures can be calculated quickly. Equation (29) represents gas flows in each pipeline to a contingency of pipeline p'. Also, (30) computes pressure squared in each node to a failure of pipeline p'. These post-contingency variables are bounded in (16) and (17) according with the secure limits:

$$\widetilde{fg}_p = fg_p + d_{p,p'}^{ng} \cdot fg_{p'} \tag{29}$$

$$\tilde{\pi}_i = \pi_i + \delta_{i,p'}^{ng} \cdot fg_{p'}. \tag{30}$$

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