



# A nonlinear control method for price-based demand response program in smart grid



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## ARTICLE INFO

### Article history:

Received 10 July 2014

Received in revised form 27 May 2015

Accepted 21 July 2015

Available online 21 August 2015

### Keywords:

Smart grid

Demand response

Power management system

Nonlinear control

Disturbances

## ABSTRACT

This paper proposes a price-based demand response program by the nonlinear control method. The demand response program is formulated as a nonlinear power management system with price feedback. We give the conditions of the price parameters for both the global asymptotic stability of the system and the social welfare optimality of the equilibrium point. Furthermore, the system is shown to be input-to-state (ISS) stable when there are additive disturbances on the power measurements and the price, and the discrete-time implementation of the power management system is given. Simulation results demonstrate the balance between supply and demand and the stability of the system with and without disturbances.

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## 1. Introduction

Smart grid is an intelligent power system that integrates advanced control, communications, and sensing technologies into the power grid [1]. In smart grid, demand response can motivate customers to shift their loads from on-peak to off-peak periods [2]. It is widely accepted that demand response is a more cost-effective way than providing enough generation capabilities to meet the peak load [3–7]. In general, there are two categories of demand response programs: incentive-based programs and price-based programs. The incentive-based programs include the direct load control program, the emergency demand response program, and the ancillary services market. For the price-based programs, the utilities can change the power consumption of customers by pricing, such as time of use (TOU), critical peak pricing (CPP), extreme day CPP (ED-CPP), extreme day pricing (EDP), and real-time pricing (RTP) [8]. Smart grid increases the opportunities for demand response by providing real-time data to providers and customers. In smart grid, the price can be provided to the customers in real time. For example, the electricity provider announces electricity prices on a rolling basis in the RTP program, and the price for a given time period (e.g., an hour) is determined

and published before the start of the period (e.g., 15 min beforehand).

There exist a number of literature on the price-based demand response programs. Different demand response programs were developed based on game theory [9–11], stochastic optimization [12,13], intelligent optimization [14], and dual decomposition method [15,16]. The social welfare maximization was achieved by optimizing the individual utilities of the customers in the demand response program based on dual decomposition. Then, a distributed power control algorithm was proposed for demand response with communication loss [17]. The works mentioned above assumed that the price is adjusted according to a pricing algorithm instead of an explicit pricing function. Recently, a linear pricing function was developed to achieve the balance between supply and demand for smart grid [18,19], and a nonlinear pricing function was used to design a distributed demand response algorithm [20]. Nevertheless, few works are devoted to the social optimality of the distributed power control under nonlinear pricing function and the influence of the disturbances on the power control algorithm.

In this study, we use a quadratic pricing function and establish the conditions on the social optimality of the distributed power control algorithm. Due to the unavoidable disturbances on power systems, we further consider the distributed power control with additive disturbances on the power measurements and the price. The differences between our work and the other smart grid algorithms are shown in Table 1. To the best of our knowledge,

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**Table 1**  
Comparisons with other smart grid algorithms.

	Pricing function	Disturbances	Social optimality
[11,18,19]	Linear	×	×
[9,20]	Nonlinear	×	×
[15,16]	×	×	✓
[17]	×	Communication loss	✓
Our work	Nonlinear	Additive errors	✓

the social optimality of the distributed power control under the nonlinear pricing function and the influence of the disturbances on the power control algorithm have not been studied. The main contributions are as follows.

- The price-based demand response program is formulated as a nonlinear power management system.
- The condition is established for the equivalence of the equilibrium point of the system and the optimal solution of a social welfare maximization problem.
- The proof of the stability is given for the power management system with and without disturbances on the power measurements and the price.

The rest of the paper is organized as follows. In Section 2, the demand response program is formulated as a nonlinear power management system. In Section 3, the conditions of the price parameters are established for both the global asymptotic stability of the system and the social welfare optimality of the equilibrium point. In Section 4, the input-to-state (ISS) stability is shown for the system with disturbances on the power measurements and the price. The discrete-time implementation of the power management system is proposed in Section 5, and the simulation results are given in Section 6. Finally, conclusions are summarized in Section 7.

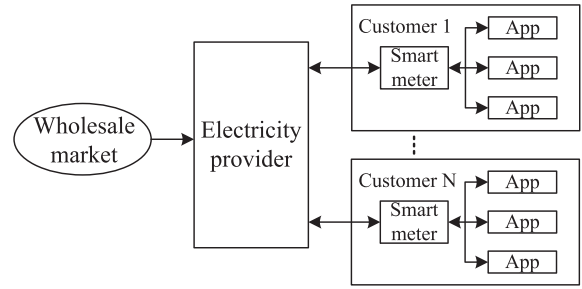
## 2. System model

As shown in Fig. 1, we consider a smart power system consisting of one electricity provider and  $N$  customers. The operation cycle of the power system is divided into several time slots. In each time slot, the electricity provider decides the electricity price and announces it to the customers. Then, the customers manage their power consumption according to the announced price.<sup>1</sup> We employ the utility functions to characterize the profits of the customers [21]. A quadratic utility function with linear decreasing marginal benefit is defined as

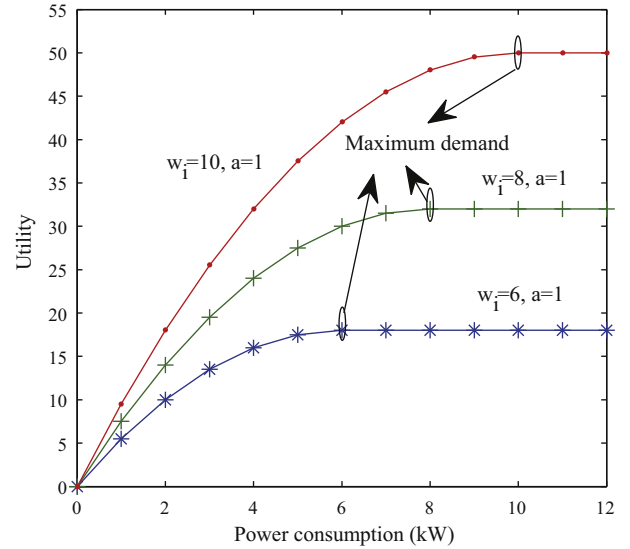
$$U_i(x_i) = \begin{cases} \omega_i x_i - \frac{a}{2} x_i^2, & \text{if } 0 \leq x_i \leq \frac{\omega_i}{a}, \\ \frac{\omega_i^2}{2a}, & \text{if } x_i > \frac{\omega_i}{a}, \end{cases} \quad (1)$$

where  $x_i$  is the power consumption of customer  $i$  ( $i \in \{1, 2, \dots, N\}$ ),  $\omega_i$  ( $\omega_i > 0$ ) denotes the willingness to increase the power consumption, and  $\omega_i/a$  denotes the maximum demand of customer  $i$ . For instance, the utility functions with different willingness parameters are shown in Fig. 2. The quadratic utility function indicates that a customer is willing to choose larger power consumption with  $\omega_i/a$  as the saturation value.

In general, the objective of demand response is to maximize the social welfare [22], which can be formulated as the following optimization problem:



**Fig. 1.** Smart power system.



**Fig. 2.** Utility functions with different willingness parameters.

$$(P1) : \begin{aligned} \max \quad & \sum_{i \in \mathcal{N}} U_i(x_i) \\ \text{s.t.} \quad & \sum_{i \in \mathcal{N}} x_i = Q, \end{aligned}$$

where  $Q$  denotes the power supply. The constraint in (P1) indicates that the total power consumption should match with the power supply. The optimization problem (P1) is a convex optimization problem and can be solved by the following primal algorithm [23]:

$$\dot{x}_i = k_i(\omega_i - ax_i - p(x)), \quad i \in \mathcal{N}, \quad (2)$$

where  $k_i$  is the control gain,  $p(x)$  is the pricing function of the electricity provider, and  $x = (x_1, \dots, x_N)^T$  denotes the set of power consumption of all the customers. In this study, we select the quadratic pricing function:

$$p(x) = b \left( \sum_{i \in \mathcal{N}} x_i \right)^2 + c \sum_{i \in \mathcal{N}} x_i, \quad (3)$$

where  $b$  and  $c$  are positive price parameters. Eqs. (2) and (3) can be integrated in a nonlinear power management system, as shown in Fig. 3.

## 3. Stability and optimality

In this section, we will study the stability of the power management system (2) and (3). Before the proof, the definition of global asymptotic stability is given.

<sup>1</sup> This assumption is for the economical theoretical behavior of the customers and is commonly used in other papers that studies price-based demand response, such as [9–11,15–20].

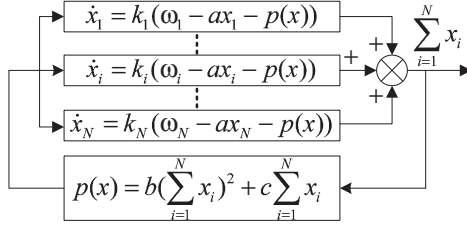


Fig. 3. Nonlinear power management system.

**Definition 1** (Stability [24]). Let  $x = 0$  be an equilibrium point for  $\dot{x} = f(x)$  with  $x(0) = x_0$ . The equilibrium point  $x = 0$  of  $\dot{x} = f(x)$  is said to be globally asymptotically stable if  $\lim_{t \rightarrow \infty} \|x(t)\| = 0$  for all initial conditions  $x_0$ .

**Theorem 1.** The power management system (2) and (3) are globally asymptotically stable if

$$a > (N - 2) \left( 2b \sum_{i \in \mathcal{N}} x_i + c \right). \quad (4)$$

**Proof.** Let  $\phi(x) = \dot{x}$ , where  $\phi(x) = (\phi_1(x), \dots, \phi_N(x))$  and  $\dot{x} = (\dot{x}_1, \dots, \dot{x}_N)$ . Define a Lyapunov candidate function as

$$V(x) = \frac{1}{2} \phi^T(x) \phi(x), \quad (5)$$

where  $V(x)$  is strictly positive for all  $x$ , except for  $x = x^*$ . The time derivative of  $V(x)$  is obtained as

$$\begin{aligned} \dot{V}(x) &= \sum_{i \in \mathcal{N}} (\phi_i(x) \cdot \dot{\phi}_i(x)) = \sum_{i \in \mathcal{N}} \left( \phi_i(x) \cdot \sum_{j \in \mathcal{N}} \frac{\partial \phi_i(x)}{\partial x_j} \cdot \dot{x}_j \right) \\ &= \sum_{i \in \mathcal{N}} \left( \phi_i(x) \cdot \sum_{j \in \mathcal{N}} \frac{\partial \phi_i(x)}{\partial x_j} \cdot \phi_j(x) \right) = \phi^T(x) J \phi(x), \end{aligned} \quad (6)$$

where  $J$  is the Jacobian matrix of  $\phi(x)$  and can be defined as

$$J = \begin{bmatrix} -a - 2b\sigma - c & -2b\sigma - c & \dots & -2b\sigma - c \\ -2b\sigma - c & -a - 2b\sigma - c & \dots & -2b\sigma - c \\ \vdots & \vdots & \ddots & \vdots \\ -2b\sigma - c & -2b\sigma - c & \dots & -a - 2b\sigma - c \end{bmatrix}, \quad (7)$$

where  $\sigma = \sum_{i \in \mathcal{N}} x_i$ . Combining with (4),  $J$  is a diagonally dominant matrix with  $J_{ii} < 0$ . Following the Gershgorin's theorem [25],  $J$  is a negative definite matrix, and the power management system (2) and (3) are globally asymptotically stable by the Lyapunov stability theorem.  $\square$

Next, we will give the conditions of the price parameters to guarantee the equivalence of the equilibrium point of the power management system and the optimal solution of (P1).

**Theorem 2.** The equilibrium point of the power management system (2) and (3) is the optimal solution of (P1) if

$$\sum_{i \in \mathcal{N}} \omega_i = NbQ^2 + (Nc + a)Q. \quad (8)$$

**Proof.** The equilibrium point of the system is obtained from the following equations:

$$\omega_i - ax_i - b \left( \sum_{i \in \mathcal{N}} x_i \right)^2 - c \sum_{i \in \mathcal{N}} x_i = 0, \quad i \in \mathcal{N}. \quad (9)$$

Adding the two sides of (9) from 1 to  $N$ , yields

$$\sum_{i \in \mathcal{N}} x_i = \frac{-(Nc + a) + \sqrt{(Nc + a)^2 + 4Nb \sum_{i \in \mathcal{N}} \omega_i}}{2Nb}. \quad (10)$$

Combining with (8), the equilibrium point of the power management system (2) and (3) satisfies

$$\begin{cases} \sum_{i \in \mathcal{N}} x_i - Q = 0, \\ \omega_i - ax_i = bQ^2 + cQ, \quad i \in \mathcal{N}. \end{cases} \quad (11)$$

Defining  $\lambda = bQ^2 + cQ$ , we have

$$\begin{cases} \sum_{i \in \mathcal{N}} x_i - Q = 0, \\ \omega_i - ax_i = \lambda, \quad i \in \mathcal{N}, \end{cases} \quad (12)$$

which is the sufficient and necessary condition for the optimality of the convex optimization problem (P1) [26], where  $\lambda$  is the Lagrangian multiplier of the following Lagrangian function:

$$L(x, \lambda, v) = \sum_{i \in \mathcal{N}} U_i(x_i) - \lambda \left( \sum_{i \in \mathcal{N}} x_i - Q \right), \quad (13)$$

Therefore, the equilibrium point of the power management system (2) and (3) is the optimal solution of (P1).  $\square$

#### 4. Power management system with additive disturbances

In reality, the power measurements and the price are not accurate due to the errors in the two-way communications between the electricity provider and the customers. It is necessary to study the impact of disturbances on the power management system. As shown in Fig. 4,  $d_1$  and  $d_2$  denote the additive disturbances on the price and the total power consumption, respectively. Then, the power control algorithm with disturbances is denoted as

$$\dot{x}_i = k_i(\omega_i - ax_i - p(x) + d_1), \quad i \in \mathcal{N}, \quad (14)$$

and the electricity price with disturbances is denoted as

$$p(x) = b \left( \sum_{i \in \mathcal{N}} x_i + d_2 \right)^2 + c \left( \sum_{i \in \mathcal{N}} x_i + d_2 \right). \quad (15)$$

Next, we study the ISS stability for the power management system with additive disturbances and denote  $p(x)$  as  $p$  for short. Before the proof, we first give the following lemma:

**Lemma 1** (ISS Stability [24]). Support that  $W : [0, \infty) \rightarrow \mathbb{R}$  satisfies

$$D^+ W(t) \leq -\alpha W(t) + \beta(t), \quad (16)$$

where  $D^+$  denotes the upper Dini derivative,  $\alpha$  is a positive constant, and  $\beta \in L_p$ ,  $p \in [1, \infty)$ . Then

$$\|W(t)\|_{L_p} \leq (\alpha \mathbf{h})^{-1/\mathbf{h}} W(0) + (\alpha \mathbf{g})^{-1/\mathbf{g}} \|\beta\|_{L_p}, \quad (17)$$

where  $\mathbf{h}$  is the complementary index of  $\mathbf{g}$ . When  $p = \infty$ , the following estimate holds:

$$\|W(t)\| \leq e^{-\alpha t} \|W(0)\| + \alpha^{-1} \|\beta\|_{L_\infty}. \quad (18)$$

Then, we obtain the following theorem:

**Theorem 3.** The power management system (14) and (15) are input-to-state stable, yields

$$\|\tilde{x}\|_{L_p} \leq \sqrt{k} (\alpha_1 \mathbf{h})^{-1/\mathbf{h}} \sqrt{\tilde{x}(0)^T K^{-1} \tilde{x}(0)} + \sqrt{2k} (\alpha_1 \mathbf{g})^{-1/\mathbf{g}} \beta_1, \quad (19)$$

$$\|\tilde{p}\| \leq \xi \sqrt{N} \|\tilde{x}\|_{L_p} + \xi d_2, \quad (20)$$

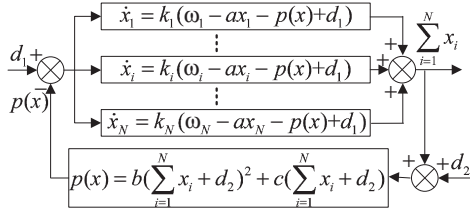


Fig. 4. Power management system with additive disturbances.

where

$$\begin{aligned} \tilde{x} &= x - x^*, \\ \tilde{p} &= p - p^*, \\ K &= \text{diag}\{k_1, \dots, k_N\}, \\ \alpha_1 &= ak, \\ \beta_1 &= \sqrt{\frac{Nk}{2}}(\xi d_2 + d_1), \\ \bar{k} &= \max_i \{k_i\}, \\ \underline{k} &= \min_i \{k_i\}, \\ \xi &= f\left(\frac{\omega}{a}\right), \quad \omega = (\omega_1, \dots, \omega_N)^T, \end{aligned}$$

and  $\mathbf{g}$  and  $\mathbf{h}$  are complementary indices, gives

$$\mathbf{g}^{-1} + \mathbf{h}^{-1} = 1,$$

when  $L_p = L_\infty$ , the system satisfies the ISS estimate

$$\|\tilde{x}(t)\| \leq \sqrt{k} \sqrt{\tilde{x}(0)^T K^{-1} \tilde{x}(0)} e^{-\alpha_1 t} + \sqrt{2k} \beta_1 / \alpha_1. \quad (21)$$

**Proof.** Consider the Lyapunov candidate function:

$$V = \frac{1}{2} \tilde{x}^T K^{-1} \tilde{x}. \quad (22)$$

The derivative of  $V$  along the trajectories of (14) is denoted as

$$\dot{V} \leq \tilde{x}^T (\omega - ax - q), \quad (23)$$

where  $q = R^T(p - d_1)$  and  $R = [1, 1, \dots, 1]_{1 \times N}$ . Adding and subtracting  $\tilde{x}^T q^*$  from the right-hand side of (23), we obtain

$$\dot{V} \leq -a\tilde{x}^T \tilde{x} - \tilde{x}^T (q - q^*). \quad (24)$$

Combining with  $q = R^T(p - d_1)$ , we have

$$\begin{aligned} \dot{V} &\leq -a\tilde{x}^T \tilde{x} - \tilde{x}^T (R^T(p - d_1) - R^T p^*) \\ &= -a\|\tilde{x}\|^2 - \tilde{x}^T R^T (f(Rx + d_2) - f(Rx^*) - d_1). \end{aligned} \quad (25)$$

Next, adding and subtracting  $\tilde{x}^T R^T f(Rx)$  from the right-hand side of (25), we obtain

$$\begin{aligned} \dot{V} &\leq -a\|\tilde{x}\|^2 - \tilde{x}^T R^T (f(Rx) - f(Rx^*)) - \tilde{x}^T R^T (f(Rx + d_2) \\ &\quad - f(Rx)) + \tilde{x}^T R^T d_1 \\ &\leq -a\|\tilde{x}\|^2 + \|R\| \|\tilde{x}\| (\xi d_2 + d_1) \\ &\leq -2akV + \sqrt{2} \|R\| \sqrt{k} (d_2 + d_1) \sqrt{V} = -2\alpha_1 V + 2\beta_1 \sqrt{V}. \end{aligned} \quad (26)$$

The inequality in (26) is obtained by the mean value theorem [27]. Setting  $W = \sqrt{V}$ , we obtain

$$D^+ W \leq -\alpha_1 W + \beta_1, \quad (27)$$

which, from Lemma 1, implies that

$$\|W\|_{L_p} \leq (\alpha_1 \mathbf{h})^{-1/\mathbf{h}} \|W(0)\| + (\alpha_1 \mathbf{g})^{-1/\mathbf{g}} \beta_1, \quad (28)$$

and

$$\|W(t)\| \leq e^{-\alpha_1 t} \|W(0)\| + \beta_1 / \alpha_1. \quad (29)$$

The inequalities (19) and (21) follow from (28) and (29), and

$$\|\tilde{x}(t)\| \leq \sqrt{2k} \|W(t)\|. \quad (30)$$

Likewise, the inequality (20) follows from the inequality

$$|\tilde{p}| \leq \xi \sqrt{N} \|\tilde{x}\| + \xi d_2, \quad (31)$$

upon taking the  $L_p$ -norm and applying the triangle inequality to the right-hand side.  $\square$

## 5. Discrete-time implementation

From the viewpoint of implementation, the discrete-time counterparts of the power control algorithm are considered. The discrete-time control algorithm without disturbances is given as

$$x_i(m+1) = x_i(m) + \mu(\omega_i - ax_i(m) - p(m)), \quad (32)$$

$$p(m+1) = b \left( \sum_{i \in \mathcal{N}} x_i(m) \right)^2 + c \sum_{i \in \mathcal{N}} x_i(m), \quad (33)$$

and the discrete-time control algorithm with disturbances is denoted as

$$x_i(m+1) = x_i(m) + \mu(\omega_i - ax_i(m) - p(m) + d_1), \quad (34)$$

$$p(m+1) = b \left( \sum_{i \in \mathcal{N}} x_i(m) + d_2 \right)^2 + c \left( \sum_{i \in \mathcal{N}} x_i(m) + d_2 \right). \quad (35)$$

In practice, the electricity provider sets the electricity price according to the forecast demand and announces the price to the customers. Each customer manages its power consumption according to the announced price. Then, the electricity provider updates the price based on the total power consumption.

The flow chart of the demand response program is shown in Fig. 5, and the program is executed with the following steps:

**Step 1:** The electricity provider sets the initial electricity price according to the forecast demand and then announces it to the customers.

**Step 2:** The customers adjust their power consumption according to (34). Defining a small positive scalar  $\delta$ , the demand response program is turned to step 3 if  $|x_i(m+1) - x_i(m)| > \delta$  for any  $i = 1, 2, \dots, N$ . Otherwise, the demand response program is terminated.

**Step 3:** The electricity provider updates the electricity price according to (35) and then announces the updated price to the customers. Then, the demand response program is turned to step 2.

The implementation of the distributed power control algorithm (34) and (35) are shown in Fig. 6. Specifically, the electricity provider collects the power consumption and announces the price to the customers based on remote meter reading system, and the additive disturbances denote the errors in the remote meter readings and published price. The data aggregate units (DAU) are deployed as gateways to forward the meter readings and the price.

## 6. Numerical results

In the simulations, we consider a residential power system composed of ten customers and one electricity provider. The power supply  $Q$  is varying from 10 kW to 42 kW. The parameters  $a$  and  $b$  are set to 3.3 and 0.01, respectively. The step size  $\mu$  is 0.07. The

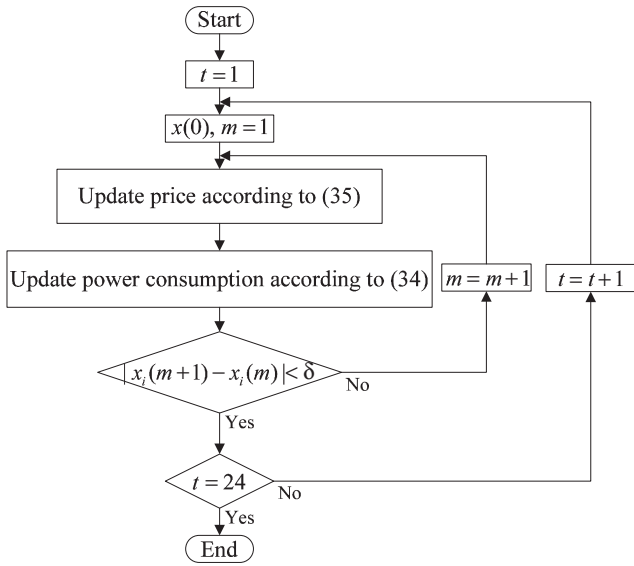


Fig. 5. Flow chart of the demand response program.

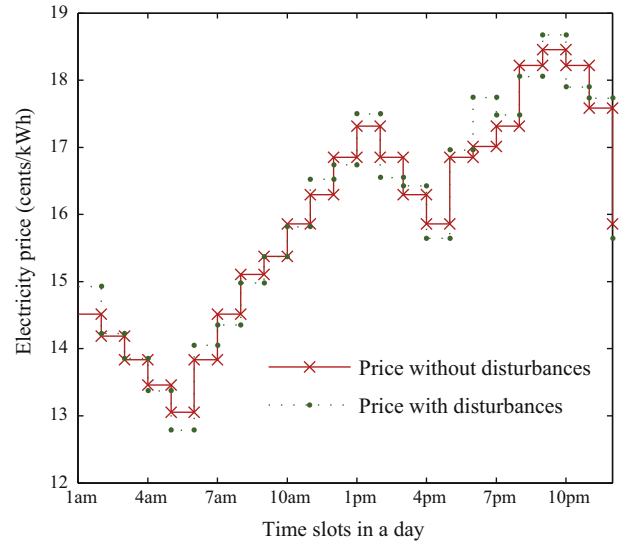


Fig. 7. Electricity price across different time slots in a day.

willingness parameter  $\omega_i$  is randomly selected from [20,25]. The electricity prices in different time slots are shown in Fig. 7 for the power management system with and without disturbances. We observe that the disturbances cause errors to the electricity price. As shown in Fig. 8, the total power consumption (TPC) matches exactly with the power supply across different time slots in a day when there are no disturbances. However, the TPC will deviate from the power supply when there exist some disturbances on the power measurements and the electricity price. It is concluded that the inaccurate price will further result in the deviations

of the TPC from the power supply. To study the impact of the disturbances on the deviations, we define the average deviation of the TPC in a day as

$$A_c = \frac{\sum_{t=1}^{24} |Q^t - \sum_{i \in N} x_i^t|}{24}, \quad (36)$$

and define the average deviation of the electricity price in a day as

$$A_p = \frac{\sum_{t=1}^{24} |p_o^t - p_d^t|}{24}, \quad (37)$$

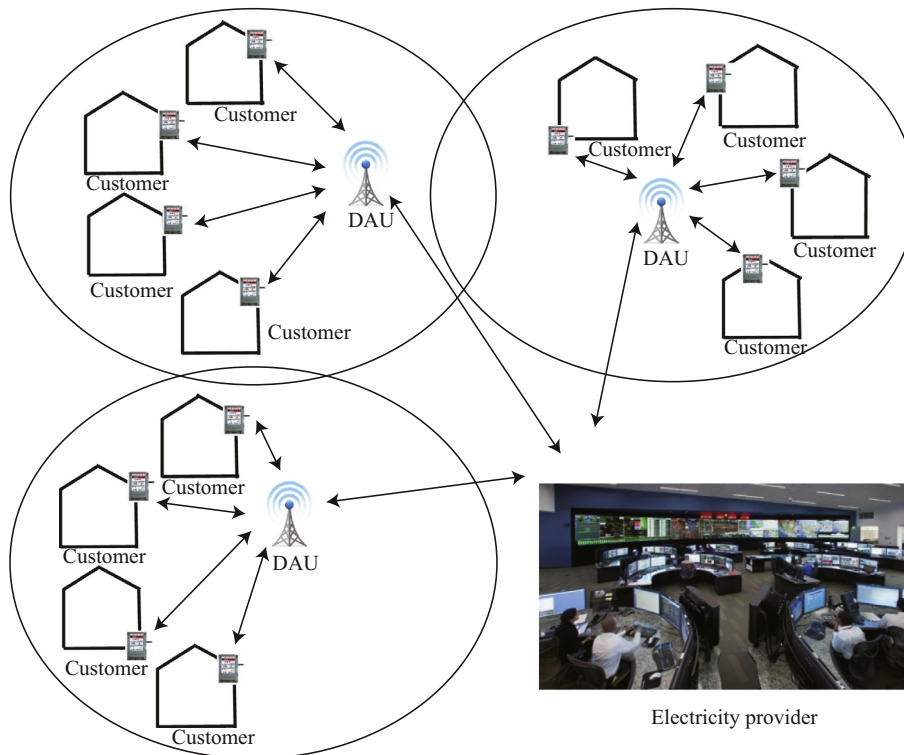


Fig. 6. Implementation in the real demand management system.

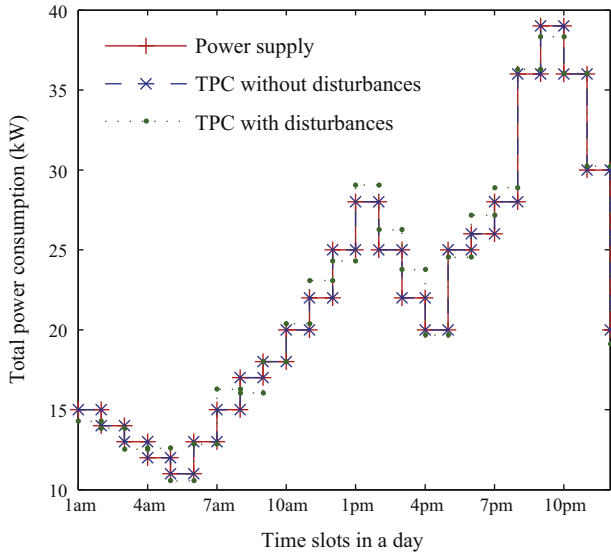


Fig. 8. Total power consumption and power supply across different time slots in a day.

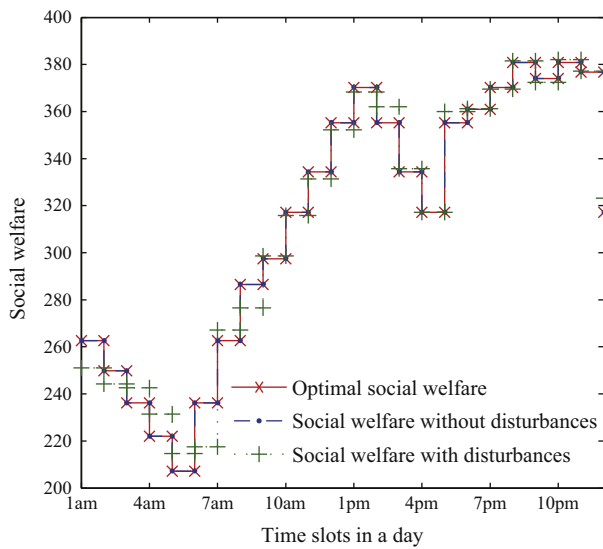


Fig. 9. Social welfare obtained from the power management system.

where  $t$  denotes the time slot in a day,  $x_i^t$  is the power consumption of customer  $i$  in time slot  $t$ ,  $p_0^t$  is the electricity price without disturbances, and  $p_d^t$  is the electricity price with disturbances. From the definitions (36) and (37), we obtain  $\Delta_c = 0.64$  and  $\Delta_p = 0.21$ .

The social welfare (i.e.,  $\sum_{i \in \mathcal{N}} U_i(x_i)$ ) obtained from the power management system is given in Fig. 9. It is shown that the social welfare obtained from the power management system achieves the optimal value of (P1), and the disturbances will result in the deviations of the social welfare from the optimal value.

To characterize the peak load shifting in a day, the peak-to-average ratio (PAR) [9] is defined as

$$\text{PAR} = \frac{24 \max_{t \in \{1, \dots, 24\}} \sum_{i \in \mathcal{N}} x_i^t}{\sum_{t=1}^{24} \sum_{i \in \mathcal{N}} x_i^t},$$

and the daily average price (DAP) is defined as

$$\bar{p} = \frac{\sum_{t=1}^{24} p^t (\sum_{i \in \mathcal{N}} x_i^t)}{\sum_{t=1}^{24} \sum_{i \in \mathcal{N}} x_i^t},$$

Table 2  
Performances of power consumption control with and without disturbances.

	DPC (kW h)	DAP (cents/kW h)	PAR
Disturbances	530.0	16.5	1.8
No disturbances	528.4	16.5	1.7

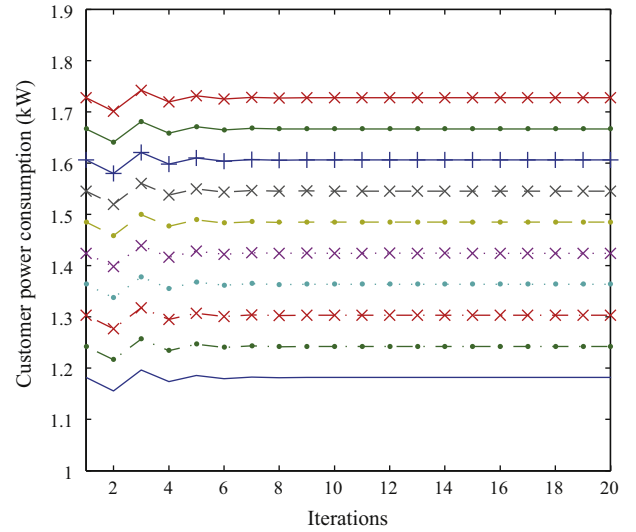


Fig. 10. Convergence of power control algorithm without disturbances.

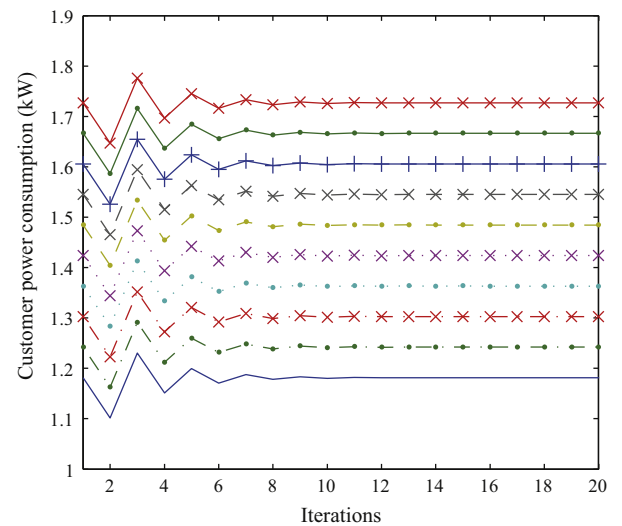


Fig. 11. Convergence of power control algorithm with disturbances.

where  $p^t$  is the electricity price in time slot  $t$ . We compare the daily power consumption (DPC), the DAP, and the PAR in Table 2. The results demonstrate that the disturbances make no change to the DAP but increase the DPC and the PAR.

The convergence of the power control algorithms without and with disturbances are shown in Figs. 10 and 11, respectively. The convergence of price in such two cases are shown in Fig. 12. It is observed that both the power consumption and the price can converge within 30 iterations. Typically, the disturbances incur longer settling time and larger overshoot in the adjustment of the power consumption and the price.

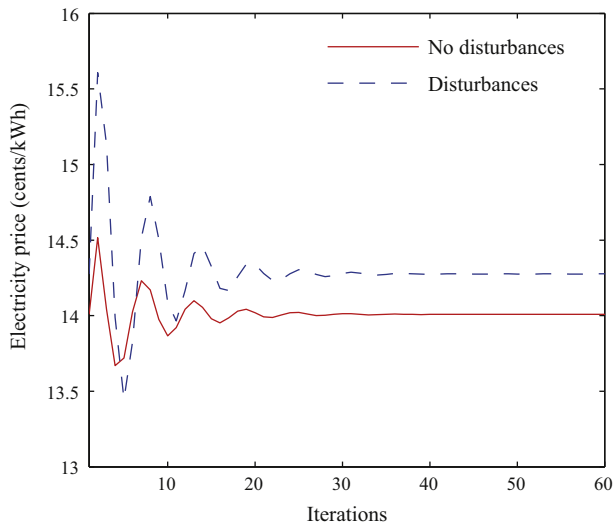


Fig. 12. Convergence of electricity price with and without disturbances.

## 7. Conclusion

This paper uses a nonlinear control method to generate a price-based demand response program. The demand response program is formulated as a nonlinear power management system, and the stability is shown for the system with and without disturbances. It is shown that the power management system can match supply with demand when there are no disturbances, and the disturbances will result in the errors in electricity price and the matching errors between supply and demand. This further degrades the transient performance of the system. In the future, we will consider the demand response program with renewable energy supplies, which will generate a stochastic power management system. Further results should be given for the stability of the stochastic system with and without disturbances.

## Acknowledgement

This work was supported in part by National Natural Science Foundation of China under Grants 61503324, 61573303, 61473202, 61172095 and 61203104 and in part by Project Funded by China Postdoctoral Science Foundation under Grant 2015M570233.

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