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A novel economic model for price-based demand response

Saeed Mohajeryami*, Iman N. Moghaddam, Milad Doostan, Behdad Vatani, Peter Schwarz

University of North Carolina at Charlotte, Charlotte, NC 28223, USA

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ABSTRACT

This paper proposes an economic model for the demand response which can explain the change and the cross-period shift in consumption pattern of individual consumers. The objective of the proposed model is to maximize the customer utility constrained by either the daily budget or daily consumption. To develop the proposed model, first, the customer utility function is described and maximized. Then, the selection of the proper parameters for the model is discussed by sensitivity analysis. Next, an algorithm is proposed to apply the model to a TOU demand response program. Subsequently, the proposed algorithm is employed to apply the model to a case study. The results demonstrate the power of the model in explaining the change and cross-period shift in consumption pattern.

1. Introduction

The electricity markets are increasingly developing over many years of restructuring and competition. However, there are still some areas in this industry kept isolated from the market's advancement one of which is demand-side. Indeed, such detachment can introduce a lot of inefficiency and inflexibility in the market. Nevertheless, recent studies demonstrate that demand response (DR) programs can break this isolation and provide an environment where the customers could actively be engaged in the optimization process and change their consumption pattern in response to the wholesale market price signals [1].

Based on the definition provided by the US department of energy, the demand response is a program established to incent the end-use customers to change their electric use from their normal consumption pattern in response to the changes in the price of the electricity over time, or to give incentive payments designed to induce lower electricity use at the times of high market prices or when the grid reliability is jeopardized [2].

The demand-side participation in electricity market could potentially create a multitude of possibilities for system operators as well as utilities to improve their system performance. These possibilities include an alternative solution for congestion management, absorbing excessive renewable generation and avoiding the extreme grid conditions in a very efficient and cost-effective manner [3]. Moreover, the demand response programs can act as a cost-effective tool for maintaining the reliability of the system while conventionally the reliable operation of the system used to require a highly capital intensive infrastructure.

Demand response programs are generally divided into two main categories of incentive-based programs (IBP) and time-based rate (TBR) programs. Each category is composed of several programs as shown in Fig. 1. In [4,5], these programs are elaborated in detail.

IBPs have been introduced to power industry for many years mostly offered to large industrial and commercial customers. For example, ERCOT offers emergency interruptible load program for large customers. Also, SCE provides a variety of demand response programs such as automated demand response (Auto-DR), permanent load shifting (PLS) and scheduled load reduction programs (SLRP) [6].

TBR programs have been historically neglected by utilities due to technical complication and highly capital intensive infrastructure. However, recently the US government and the energy sector, in response to the environmental challenges of the traditional electricity generation, have adopted a very supportive approach to create the required infrastructure for the demand response and energy efficiency programs including TBR programs.

According to 2010 FERC survey, advanced metering penetration which is a cornerstone of TBR programs has been reached approximately to a considerable level of 8.7 percent in the US [7]. Moreover,

* Corresponding author. Tel.: +1 7049186334.
E-mail addresses: smohajer@uncc.edu (S. Mohajeryami), inazirim@uncc.edu (I.N. Moghaddam), mdoostan@uncc.edu (M. Doostan), bvatani@uncc.edu (B. Vatani), pschwarz@uncc.edu (P. Schwarz).

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Nomenclature

The main notations used throughout the paper are stated below for quick reference. Other symbols are defined as required throughout the text.

$U(.)$ consumer utility
C_{peak} share of the electricity consumption of each household at peak time
$C_{\text{off peak}}$ share of the electricity consumption of each household at off-peak time
ρ consumption proportion
θ control for the substitution effect
p_{peak} price of the peak time electricity
$p_{\text{off peak}}$ price of the off peak time electricity
I daily budget
IP maximum electricity possible to consume at peak time with the daily budget
r relative price of the peak time compared to the off peak time

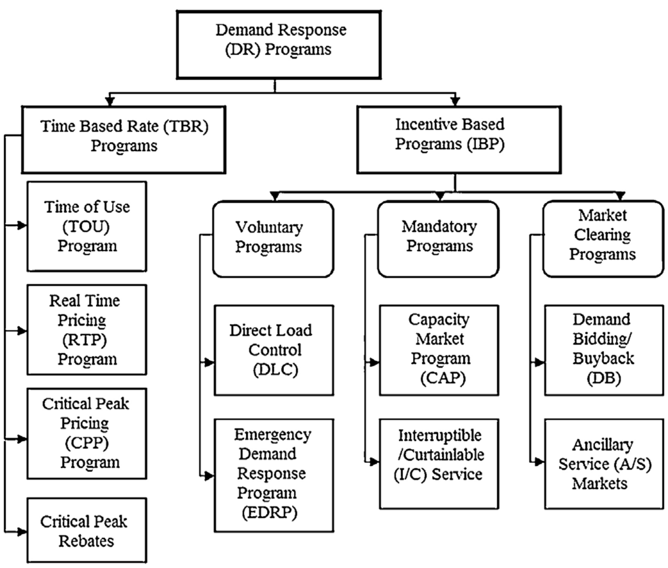

Fig. 1. Categories of the demand response programs.

many utilities have recently launched pilot programs to study the feasibility and technical challenges of TBR programs [8–12].

While the availability of financial support and necessary infrastructure have facilitated utilizing TBR programs, the implementation of such programs may face sundry obstacles in practice. One of the most considerable issues for utilities in this regard is finding the proper model to explain the customer's response to these programs. Indeed, in the absence of a reliable model, the utilities cannot employ the proper profit maximizing strategies, which leads to losing the incentive to participate in the program. Therefore, to overcome this problem, different models are proposed to describe the consumer behavior under time-based programs.

Authors in [13–15] employ demand–price elasticity concept to model the customer's reaction in demand response programs. In order to understand their proposed model, it is necessary to understand the concept of demand–price elasticity. In fact, this concept is borrowed from the consumer choice theory in microeconomics. The demand for almost all goods and services rises as the price decreases. The change in the demand is not linear according to diminishing marginal rate of return law. To quantify the aforesaid change in the demand, demand–price elasticity is to be utilized. Indeed, the nonlinear demand curve could be linearized around any given point. The change in the demand relative to the change in price is known as demand–price elasticity [16,17].

Authors in [18] model the consumer's behavior by using a matrix of self- and cross-elasticities. They analyze the effect of market structure on the elasticity of demand for electricity. Later, many researchers tried to expand this method. Price–demand elasticity is also utilized in [19] for creating a model for combined TOU and emergency demand response (EDR) program. In [20], the inverse electricity rule is applied as a basis of TOU pricing. The own- and cross-elasticity is determined by the regression on the experimental data of TOU program in Taiwan for the sectoral demand elasticity. Moreover, in [21], a statistical method is introduced which uses the demand elasticity in the context of Direct Load Control (DLC) program. This paper employs a statistical iterative procedure to generate a parametric customer utility function which explains the price elastic behaviors of aggregated loads through a set of multi-dimensional demand–price functions. Additionally, the developed model in this paper is utilized to determine the optimal price signal for Real-Time Pricing (RTP) based DR programs.

One of the shortcomings of price–demand elasticity employed in the aforementioned papers is that every value is only valid for a certain point on the price–demand curve. Therefore, every price–demand elasticity value explains the customer's demand response for a certain circumstance; hence, in case of any changes, this value should be recalculated with new set of observations.

In addition to challenges introduced by modeling of demand response, there are some arguments on how to determine price–demand elasticity of electricity. Authors in [22] compare and summarize the results of more than 15 published papers working on the electricity elasticity to classify and determine the short and long term elasticities. Moreover, authors in [23] review all the studies done in the area of electricity price elasticity. This paper utilizes the historical data from South Australian electricity demand to calculate the price elasticity of electricity demand in South Australia. The price elasticity, as discussed in [23] is different for each industrial section or load cluster. It, also, varies over time and in response to the introduction of more efficient technologies. Therefore, it is necessary to have the empirical data from the target area to extract the price–demand elasticity. Many demand response pilot projects are launched by different utilities all over the US [21,24–28] to study the customer behaviors subject to the varying prices. Many of these pilot project utilized the data to extract the relevant price–demand elasticity to determine the characteristics of retail electricity demand.

In addition to the models introduced so far, some other papers propose approaches for understanding the customer's reaction which do not rely on any deterministic model as opposed to price–demand elasticity based models. One of the shortcomings of such approaches is heavy computation required to examine a multiple-scenario decision as opposed to the deterministic models. In [24], an algorithm is proposed which attempts to describe the real time demand response with real time optimization. This paper proposes an optimization model to adjust the customer's hourly load in response to hourly electricity prices. The goal of the proposed approach is to maximize the utility of the consumer. In a similar attempt [29], tries to incorporate the effect of DR participation on the market price. It proposes an optimization problem that attempts to consider both different DR price elasticity characteristics and DR participation levels to converge to a market clearing price in a volatile power market. In order to solve this optimization problem, this paper employs a closed-loop iterative simulation method and a non-iterative method based on the contraction mapping theorem. In this paper, convergence refers to the fact that load and/or price values will finally converge to a fixed point after a finite number of iterations between the economic dispatch problem and the price-sensitive DR load adjustment. Moreover, authors in [30], similar to [24], try to calculate demand response by real time optimization. However, what distinguishes this paper from the earlier work is that the major appliances of a typical household such as Electric Vehicle (EV), Electric Water Heater (EWH), HVAC, refrigerator and their interaction with utility are modeled separately and integrated into the optimization model.

In another attempt [31], tries to model demand response in a dynamic manner by employing Monte Carlo simulation. This paper presents a demand response model with elastic economic dispatch in a locational marginal pricing market. It models system economic dispatch as a feedback control process, and introduces a flexible and adjustable price as a controlled signal to adjust demand response. One advantage of this approach is the fact that a model of electricity market is included in the proposed approach which makes it more realistic; however, due to many uncertainties in electricity market and demand-side behavior, it heavily relies on many assumption which makes it oversimplified.

Moreover, authors in [3] proposed a model which unlike all the aforementioned models and approaches does not need the price–demand elasticity or demand forecast. In this model load aggregator asks the customers to submit their candidate load profiles ranked in the preference order. Then, the load aggregator performs the final selection of individual load profiles subject to the total system cost minimization. Although this approach requires only information about user's daily consumption needs and preferences, it limits the customers significantly in terms of availability of options and flexibility. In practice, it is very difficult for the customers to know their accurate future load profile beforehand.

In DR models discussed so far, the underlying assumption is that the people who participate in the program are reasonable. In other words, they try to maximize their benefits and for carrying out such intention, they are willing to do all the necessary accommodations. However, there are several works suggesting that this underlying assumption should be revisited. This discussion roots in behavioral economics. These works argue that there are many behavioral parameters which affect the customer's decision that must be included in the model. In [32], it is discussed that the customers' reactions to the reward and punishment incentives are different and since DR programs differ in this regard. Therefore, given the nature of DR programs, the customers could under- or overreact. Consequently, this behavioral parameter should be included in any model attempts to examine the customer's reaction. Moreover, [33] argues that people are loss-averse and if this behavioral parameter is neglected in the model, the results are completely misleading. Since the behavioral studies of DR programs are in the early stage, such behavioral parameters are not considered in this paper.

Moreover, the results of the proposed models in the literature are to be validated by the observations from the real observations acquired from the experimental projects. For carrying out such tasks, the characteristics of demand response which are observed and studied in different works are reviewed in this section.

Authors in [10] present an experience from a pilot study in Trondheim, Norway. The pilot project focuses on daily demand response from the households in Trondheim. In this project the data come from the smart meters; also, for the pricing data, a combination of hourly spot price and TOU tariff is utilized. The results of the experiment provide a framework for comparison which could be employed in any potential model of TOU demand response. In other words, if the results of the proposed model resemble the real observations, it is fair to conclude that the model is a good enough. Inspired by [10], the same authors, suggest a project for European Union (EU) to provide additional balancing resources from flexible consumption. The DR model employed by this paper relies on the price elasticity to determine model parameters [34].

In [35], real observations from 163 pricing treatments offered on an experimental or full-scale basis in 34 projects are pooled in a comprehensive study. This paper provides a strong comparison tool for theoretical models to evaluate their performance. Moreover, it is found that the demand response expressed as a function of the peak to off-peak price ratio is fairly consistent and robust. The response curve is nonlinear and is shaped in the form of an arc. According to the aforementioned argument about peak to off-peak price ratio, this paper suggests that the demand response models should select this ratio as their main parameter. In our work, this suggestion is taken seriously and the peak to off-peak price ratio is employed as a main parameter. It lets the model's results be compared directly to the results of the aforementioned work.

In this paper, an economic model is proposed to explain the demand response in price-based DR programs. The goal of most of price-based DR programs is to induce customers to shift their consumption between different time periods. In this model, for the purpose of simplicity, the time periods are limited to peak-time and off-peak time. However, this model has this ability to be upgraded to incorporate as many time periods as necessary. Moreover, in order to numerically simulate the model, the proposed algorithm is applied to Time of Use (TOU) program. The reason for selecting such DR program is that this program typically offers a cheap price for the low consumption periods (off-peak time) and a high price for the high consumption periods (peak time). Therefore, since the proposed model uses only two consumption periods, this program fits perfectly to the model.

Also, the proposed model in this paper is inspired by an overlapping generation (OLG) model called Diamond's OLG model. This name is taken after the American Nobel prize laureate economist Peter A. Diamond [36,37]. The OLG model is a critical model in Macroeconomics to explain economic growth.

The OLG is originally proposed to explain the questions associated with public debt problems, taxation of capital income and financing of social security. However, it could be utilized by any economist to answer questions about the adjustment in the consumption between two time periods in response to the relative interest rate. In this paper, adjustment in the consumption between two periods of peak and off-peak in response to the change in the relative peak and off-peak price is explained by OLG model.

What distinguishes this model from the other proposed models in the literature is twofold. First, this model has this capability to incorporate the willingness of customers to reallocate the consumption cross-periods. This willingness is different for each customer and cannot be explained only by price-elasticity. Therefore, this characteristics makes this model ideal to apply to residential customers. As it will be discussed in the future sections of this paper, by analyzing the consumption history of each customer and her response to DR events, this parameter can be extracted for each customer. Second, this model reflects the classic economic decision making process of each individual. Therefore, it can easily be integrated into real time demand response models employed by utilities to (1) offer optimal dynamic pricing to customers and (2) select the optimum bidding strategy and planning. Furthermore, unlike the other models, this model explains the underlying mechanism of the shift in the consumption and provides more intuitive sense about the decision making process of each customer.

The organization of this paper is as follows. It starts with the description of the proposed model in Section 2. Then, it presents the sensitivity analysis of the model which is necessary for the proper selection of the parameters in the implementation section. Afterwards, Section 3 explains the implementation of the model. Then, Section 4 presents the results of a case study and discusses the results. Section 5 closes the paper with drawing conclusion from the provided discussion and results.

2. Demand response model

2.1. Utility function

The main goal of this paper is to provide a mathematical model which can explain how the customers react to the different electricity prices. This model can be employed by utilities to set forth certain prices for peak and off-peak times to induce customers to lower or shift their consumption to a certain level. The change of load to a certain level is aligned with the goal of simultaneously reducing the transmission congestion, Locational Marginal Price (LMP) spikes and transmission line power loss [31].

Inspired by Diamond's OLG model [36,37], the proposed model in this work attempts to explain the demand response in the framework of the consumer choice theory. For this purpose, it is necessary to discuss about the customer's utility function.

The first step to create DR model is to define the customer's utility function. The term "utility" in the economic literature refers to all the benefits, tangible or intangible, received by the customer. Utility is a measure of well-being or the level of "satisfaction" or "happiness" that a consumer obtains from various goods. Utility function is a tool to describe this level.

Assuming that this function for electricity consumption is constant relative risk aversion (CRRA), based on Diamond growth model, (1) is used to explain the customer's utility.

$$U(C)=\frac{C^{1-\theta}}{1-\theta},\quad \theta>0 \tag{1}$$

The utility function introduced as (1) uses consumption as a measure of the customer's preferences which is one of the standard forms of quantifying the preferences. There are a few other prevalent utility functions employed in microeconomics which consider some of the behavioral characteristics of consumers (e.g. risk aversion) in form of deterministic or probabilistic function. (1) is a deterministic model which allows enough flexibility to capture the substitution effect.

As for economic point of view, an acceptable utility function must have two properties, namely, positive marginal return (first derivative of utility function) and diminishing (negative) marginal rate of return (second derivative of utility function). These properties of (1) are described in (2) and (3), respectively.

$$\frac{\partial U}{\partial C}=C^{-\theta}>0 \tag{2}$$

$$\frac{\partial^2 U}{\partial C^2}=-\theta C^{-\theta-1}<0 \tag{3}$$

where θ, *theta*, controls the elasticity of the substitution between the future and current consumption. The elasticity of the substitution could be explained as the willingness of customers to reallocate the consumption cross-periods in response to change in the relative price of peak and off-peak time prices.

The customer's consumption could be divided into two periods of high consumption (peak time) and low consumption (off-peak time). The utility function for each period is different; therefore, the model is to consider separate functions for each one. The daily utility is expressed by summation of these two aforesaid functions as in

$$U(C)=\frac{C_{\text{peak}}^{1-\theta}}{1-\theta}+\frac{C_{\text{off peak}}^{1-\theta}}{(1-\theta)(1+\rho)},\quad \theta>0,\quad \rho>-1 \tag{4}$$

where C_{peak} is the share of the electricity consumption of each household at peak time, $C_{\text{off peak}}$ is the share of the electricity consumption of the same household at off-peak time, ρ, *rho*, consumption proportion factor, is a ratio of peak and off-peak consumption.

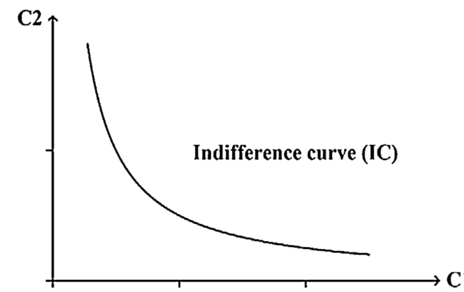

Fig. 2. Indifference curve for C1 and C2.

2.2. Substitution effect

In the utility function, θ is claimed to be a control for substitution effect. In microeconomics, the "substitution effect" is the effect due only to the relative price change, controlling for the change in real income. In order to compute it, this question should be asked that what is the other bundle of goods that make the consumer just as happy as before the price change, but if they had to make their choice faced with the new prices [38]. In what follows, the aforementioned claim about θ is going to be examined by the indifference curve (IC). The indifference curve for this utility function is as shown in Fig. 2. In microeconomics theory, the IC is a graph that every point on it has the same utility value.

Since every point on the IC has the same value, the derivative of the utility function for all the points on the utility curve is zero. Taking the derivative from the utility function gives

$$\mathrm{d}U=C_{\text{peak}}^{-\theta}.\mathrm{d}C_{\text{peak}}+\frac{C_{\text{off peak}}^{-\theta}}{(1+\rho)}.\mathrm{d}C_{\text{off peak}}=0 \tag{5}$$

(5) can be rearranged to give

$$\frac{\mathrm{d}C_{\text{off peak}}}{\mathrm{d}C_{\text{peak}}}=-\left(\frac{C_{\text{off peak}}}{C_{\text{peak}}}\right)^{\theta}.(1+\rho) \tag{6}$$

The marginal rate of substitution (MRS) between consumption at peak time and off-peak time is expressed as

$$\text{MRS}=\left(\frac{C_{\text{off peak}}}{C_{\text{peak}}}\right)^{\theta}.(1+\rho) \tag{7}$$

Taking natural log from both sides of (7) gives

$$\ln(\text{MRS})=\theta.\ln\left(\frac{C_{\text{off peak}}}{C_{\text{peak}}}\right)+\ln(1+\rho) \tag{8}$$

Taking derivative from (8) gives

$$\mathrm{d}[\ln(\text{MRS})]=\theta.\mathrm{d}\left[\ln\left(\frac{C_{\text{off peak}}}{C_{\text{peak}}}\right)\right] \tag{9}$$

And finally (9) can be rearranged to give

$$\frac{\mathrm{d}\left[\mathit{ln}\left(C_{\text{off peak}}/C_{\text{peak}}\right)\right]}{\mathrm{d}[\mathit{ln}(\mathit{MRS})]}=\frac{1}{\theta} \tag{10}$$

The last formula is the basis of the utility function for electricity consumption which simply says the rate of change of consumption between two periods to the change of the marginal rate of substitution is constant and dependent on θ.

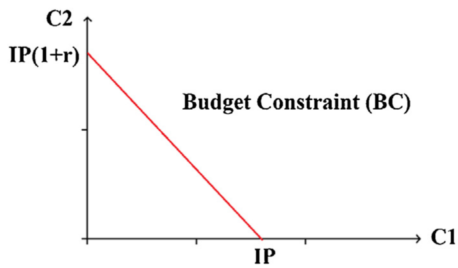

Fig. 3. Budget constraint.

2.3. Household budget

The utility is constrained by the budget. In this paper, it is assumed that every consumer has a fixed budget for daily electricity consumption. The budget constraint is given by

$$I = p_{\text{peak}}.C_{\text{peak}} + p_{\text{off peak}}.C_{\text{off peak}} \tag{11}$$

Assuming $p_{\text{peak}} \geq p_{\text{off peak}}$, (11) can be rewritten as

$$p_{\text{peak}} = (1+r).p_{\text{off peak}} \tag{12}$$

$$r = \frac{p_{\text{peak}} - p_{\text{off peak}}}{p_{\text{off peak}}} \tag{13}$$

(14) and (15) are obtained by plugging (12) and (13) into (11).

$$\frac{I}{p_{\text{peak}}} = C_{\text{peak}} + \frac{p_{\text{off peak}}}{p_{\text{peak}}}.C_{\text{off peak}} \tag{14}$$

$$IP = C_{\text{peak}} + \frac{1}{1+r}.C_{\text{off peak}} \tag{15}$$

Fig. 3 shows the budget constraint. C_1 and C_2 in this figure represent peak and off-peak consumption, respectively. If a consumer decides to spend all her budget to consume at the peak time, she can have *IP* amount of electricity and on the contrary, if she decides to spend all her budget to consume at the off-peak time, she can have $IP(1+r)$ amount of electricity given the price difference, r, and the total budget, I.

2.4. Optimization

Every individual maximizes the utility subject to the budget constraint. In Fig. 4 this concept is illustrated. In this figure, C_1^* and C_2^* are the points where indifference curve (i.e. IC) intercepts with the budget constraint curve (i.e. BC). In other words, these points are the optimal points that give the maximum utility given the budget constraint.

The individual's maximization problem can be rigorously solved by using Lagrangian multiplier method. (16) presents the setup for Lagrangian.

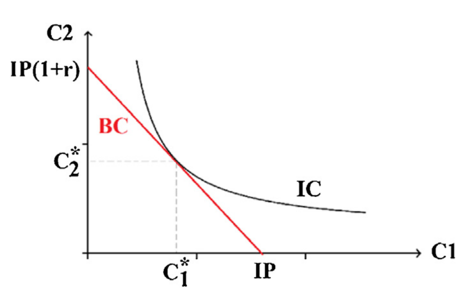

Fig. 4. Optimal consumption point.

$$\begin{aligned} L\left(C_{\text{peak}}, C_{\text{off peak}}\right) &= \frac{C_{\text{peak}}^{1-\theta}}{1-\theta} + \frac{C_{\text{off peak}}^{1-\theta}}{(1-\theta)(1+\rho)} \\ &\quad + \lambda.\left[IP - C_{\text{peak}} - \frac{1}{1+r}.C_{\text{off peak}}\right] \end{aligned} \tag{16}$$

where λ is called the Lagrangian multiplier. The first-order necessary conditions (FONC) for (16) are given by

$$\text{for } C_{\text{peak}}: \frac{\partial L}{\partial C_{\text{peak}}} = C_{\text{peak}}^{-\theta} - \lambda = 0 \tag{17}$$

$$\text{for } C_{\text{off peak}}: \frac{\partial L}{\partial C_{\text{off peak}}} = \frac{C_{\text{off peak}}^{-\theta}}{(1+\rho)} - \frac{\lambda}{(1+r)} = 0 \tag{18}$$

$$\text{for } \lambda: \frac{\partial L}{\partial \lambda} = IP - C_{\text{peak}} - \frac{1}{1+r}.C_{\text{off peak}} = 0 \tag{19}$$

(17) and (18) can be rearranged to give

$$\lambda = C_{\text{peak}}^{-\theta} = \frac{C_{\text{off peak}}^{-\theta}}{(1+\rho)} \cdot (1+r) \tag{20}$$

By taking λ out of the equation, it can be claimed that what remains is the pure substitution effect as λ is the only variable in FONCs which represent "income effect". In the context of economic theory, the "income effect" is the change in an individual's or economy's income and how that change will impact the quantity demanded of a good or service [38]. The Euler equation for the pure substitution effect in this case is

$$C_{\text{off peak}} = C_{\text{peak}} \cdot \left(\frac{1+r}{1+\rho}\right)^{1/\theta} \tag{21}$$

With having FONCs and the pure substitution effect, the consumption problem can be solved as

$$dU = C_{\text{peak}}^{-\theta}.dC_{\text{peak}} + \frac{C_{\text{off peak}}^{-\theta}}{(1+\rho)}.dC_{\text{off peak}} \tag{22}$$

Taking derivative from (15) gives

$$d[IP] = dC_{\text{peak}} + \frac{1}{1+r}.dC_{\text{off peak}} \tag{23}$$

Plugging (17) and (18) into (22) yields

$$\begin{aligned} dU &= \lambda.dC_{\text{peak}} + \frac{\lambda}{(1+r)}.dC_{\text{off peak}} \\ &= \lambda.\left[dC_{\text{peak}} + \frac{1}{(1+r)}.dC_{\text{off peak}}\right] \end{aligned} \tag{24}$$

The "income effect" on consumption can be demonstrated by plugging (23) into (24).

$$dU = \lambda.d[IP] \rightarrow \frac{dU}{d(IP)} = \lambda \tag{25}$$

Using (19) and (21), the consumption for two periods is solved as

$$\begin{aligned} IP &= C_{\text{peak}} + \frac{1}{1+r}.C_{\text{peak}} \cdot \left(\frac{1+r}{1+\rho}\right)^{1/\theta} \\ &= C_{\text{peak}} \cdot \left[1 + \frac{1}{1+r} \cdot \left(\frac{1+r}{1+\rho}\right)^{1/\theta}\right] \end{aligned} \tag{26}$$

(26) can be rearranged in the following form.

$$C_{\text{peak}} = \frac{IP}{1 + \left((1+r)^{(1/\theta)-1}/(1+\rho)^{1/\theta}\right)} \tag{27}$$

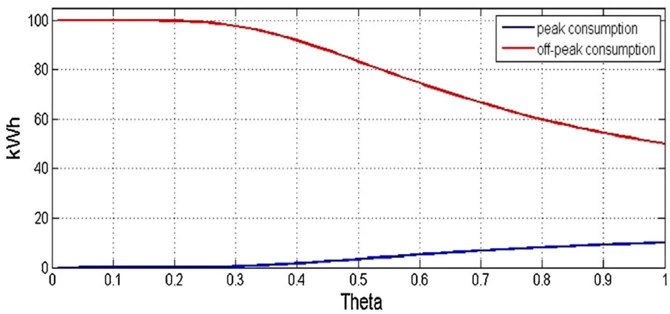

Fig. 5. Sensitivity of consumption to change of theta.

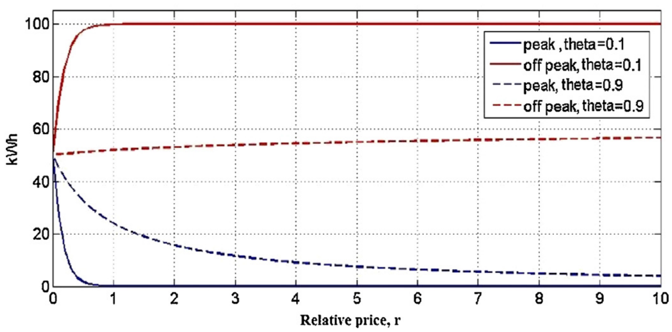

Fig. 6. Sensitivity of consumption to change of the relative price.

Consequently, $C_{\text{off peak}}$ is found as

$$C_{\text{off peak}} = \frac{IP(1+r)}{1+\left((1+\rho)^{1/\theta}/(1+r)^{(1/\theta)-1}\right)} \tag{28}$$

The power of $(1+r)$ in (27) and (28) has two components, $1/\theta$ is the "substitution effect" and -1 is an "income effect". If $1/\theta > 1$, then "substitution effect" is dominant; therefore, with increasing the relative price, the consumption at peak time will simultaneously decrease and be shift to the off-peak time. If $1/\theta < 1$, then "income effect" dominates.

2.5. Sensitivity analysis

In order to study the applications of the proposed model, proper selection of its parameters (*theta*, r and *rho*) is a necessity. To make such selection, it is essential to perform the sensitivity analysis. The sensitivity analysis is a tool to determine how under a given set of assumptions, different values of an independent variable will impact a particular dependent variable.

2.5.1. Change of theta, θ

Assuming a household has a daily budget of I = \$10 and p_{peak} = \$0.5, $p_{\text{off peak}}$ = \$0.1 (relative price is r = 4) are the price of peak and off-peak electricity. The effect of change in θ from $0.01 \rightarrow 1$ on C_{peak} and $C_{\text{off peak}}$ are depicted in Fig. 5 θ = 0.01 is a point that "substitution effect" dominates while θ = 1 is a point that the "substitution effect" and "income effect" wash each other's effect.

2.5.2. Change of the relative price, r

To examine the effect of relative price on consumption, the changes in r from $0 \rightarrow 10$ is examined on two separate cases of θ = 0.1 and θ = 0.9. The result is shown in Fig. 6. In this model, "r" can be interpreted as a parameter which explains the economic incentive of the household. The bigger "r" is, the bigger is the economic incentive.

2.5.3. Change of rho, ρ

To examine the effect of consumption proportion factor on consumption, the changes in ρ from $0 \rightarrow 1$ is examined on the case that r = 0 (relative price is 1) and θ = 0.9. The selection of theta in this case is arbitrary and is just for the illustrative purpose. The negative values of the depreciation factor are more applicable in "demand response", because the demand of the electricity is higher in the peak time. The results are shown in Fig. 7.

3. Model implementation

The proposed model needs an assumption about the consumption habits of the customers. In the framework of the consumer theory, it is argued that there is a competition between budget and the consumption when the price changes. Two extreme cases are examined in this paper. In the first case, the budget is fixed and the consumption should accommodate the new change in the price. In the second case, the consumption remains unchanged and the budget should accommodate the aforesaid change.

Fig. 8 is a flowchart illustrating the process of calculating the final load profile. It starts with loading the daily load profile. Then, it computes the peak and off-peak load. Afterwards, the peak and off-peak prices and a daily budget inputs the program. Then, it computes the peak and off-peak consumption based on the proposed model. Next, the program checks the proportion of the peak to off-peak consumption. If there is a mismatch, it adjusts the consumption proportion to reflect the peak and off-peak consumption proportion of the load profile. Then based on the selected scenario, it runs one of the following algorithms:

Fixed daily non-base load budget algorithm
1. Compute the daily non-base load budget in flat rate pricing (B1)
2. Compute the non-base peak $\left(C_{\text{peak}}^{\text{flat}}\right)$ and off-peak $\left(C_{\text{off peak}}^{\text{flat}}\right)$ load of the sample load profile
3. Use the model, apply TOU pricing and compute the non-base peak $\left(C_{\text{peak}}^{\text{TOU}}\right)$ and off-peak $\left(C_{\text{off peak}}^{\text{TOU}}\right)$ load with the budge of step 1
4. Compute the daily non-base load budget in TOU pricing (B2)
5. Check the (B1–B2), if greater than zero, increase Rho by 0.01 and go to step 3
6. Check the (B1–B2), if smaller than zero, decrease Rho by 0.01 and go to step 3
7. Check the (B1–B2), if it's equal to zero, stop and go to the next step
8. Adjust the peak and off-peak load proportionate to $C_{\text{peak}}^{\text{TOU}}/C_{\text{peak}}^{\text{flat}}$ and $C_{\text{off peak}}^{\text{TOU}}/C_{\text{off peak}}^{\text{flat}}$, respectively

Fixed daily non-base load consumption algorithm
1. Compute the non-base peak $\left(C_{\text{peak}}^{\text{flat}}\right)$ and off-peak $\left(C_{\text{off peak}}^{\text{flat}}\right)$ load of the sample load profile
2. Compute the daily non-base load consumption in flat rate pricing (C1)
3. Use the model, apply TOU pricing and compute the non-base peak $\left(C_{\text{peak}}^{\text{TOU}}\right)$ and off-peak $\left(C_{\text{off peak}}^{\text{TOU}}\right)$ load with a pre-selected budge
4. Compute the daily non-base load consumption in TOU pricing (C2)
5. Check the (C1–C2), if greater than zero, decrease the budget (I) by 0.1 and go to step 3
6. Check the (C1–C2), if smaller than zero, increase the budget (I) by 0.1 and go to step 3
7. Check the (C1–C2), if it is equal to zero, stop and go to the next step
8. Adjust the peak and off-peak load proportionate to $C_{\text{peak}}^{\text{TOU}}/C_{\text{peak}}^{\text{flat}}$ and $C_{\text{off peak}}^{\text{TOU}}/C_{\text{off peak}}^{\text{flat}}$, respectively

4. Case study

4.1. Load profile

Table 1 contains the data for a sample aggregated residential customers' load profile under the flat rate tariff of NorthWestern Energy utility [39]. Fig. 9 shows the sample load profile.

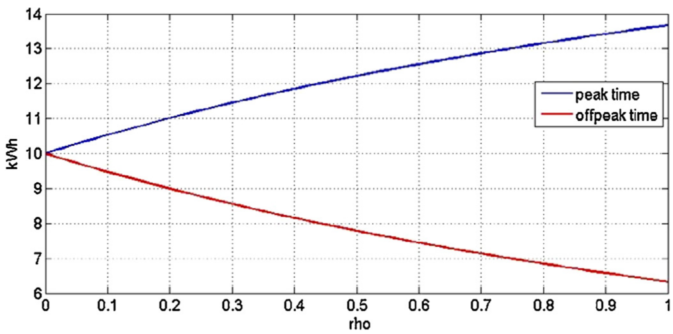

Fig. 7. Sensitivity of consumption to consumption proportion factor.

Consumption in peak time is structurally different from the off-peak time. For instance, Intensive lighting consumption in the evening, consumers' tendency to cook, watching TV after work/school, and at the same time, if it is Winter/Summer, the HVAC is in full-effect since everyone is home and these are major parts of the peak time consumption while these activities and consumption are pretty much non-existent at late night or during the day time. Due to this differences stemming from the natural structure of living in every society, there is an asymmetry between peak and off-peak consumption. Peak consumption per hour (PCPH) is bigger than off-peak consumption per hour (OPCPH). For the load profile of Fig. 9, PCPH/OPCPH = 1.36. This structural asymmetry is explained with consumption proportion, ρ, in the model.

4.2. Results and discussion

This section demonstrates the results of the model for two scenarios. These scenarios are (a) fixed budget and (b) fixed consumption for non-base loads. Arguably, the customers can adopt any one of these scenarios based on their personal preferences. In practice, for aggregate loads, it can be claimed that the demand lies somewhere between these two scenarios.

Table 1
Sample load profile.

Hour	kW h	Hour	kW h	Hour	kW h	Hour	kW h
1	5.545	**7**	4.516	**13**	9.057	**19**	9.618
2	4.975	**8**	5.587	**14**	8.674	**20**	9.116
3	4.414	**9**	6.531	**15**	8.929	**21**	8.819
4	4.176	**10**	7.458	**16**	8.657	**22**	9.04
5	4.167	**11**	9.524	**17**	9.252	**23**	8.504
6	4.431	**12**	9.116	**18**	9.916	**24**	6.625

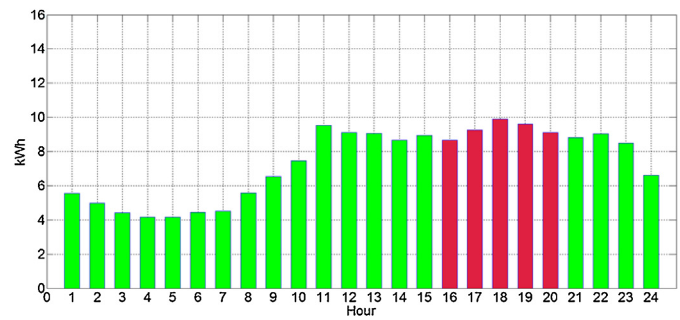

Fig. 9. Sample load profile (peak time and off-peak time are color coded with red and green, respectively).

4.2.1. Scenario 1 (fixed non-base-load budget)

A big portion of the load is almost inelastic within the current range of the electricity price. This inelastic portion is different for

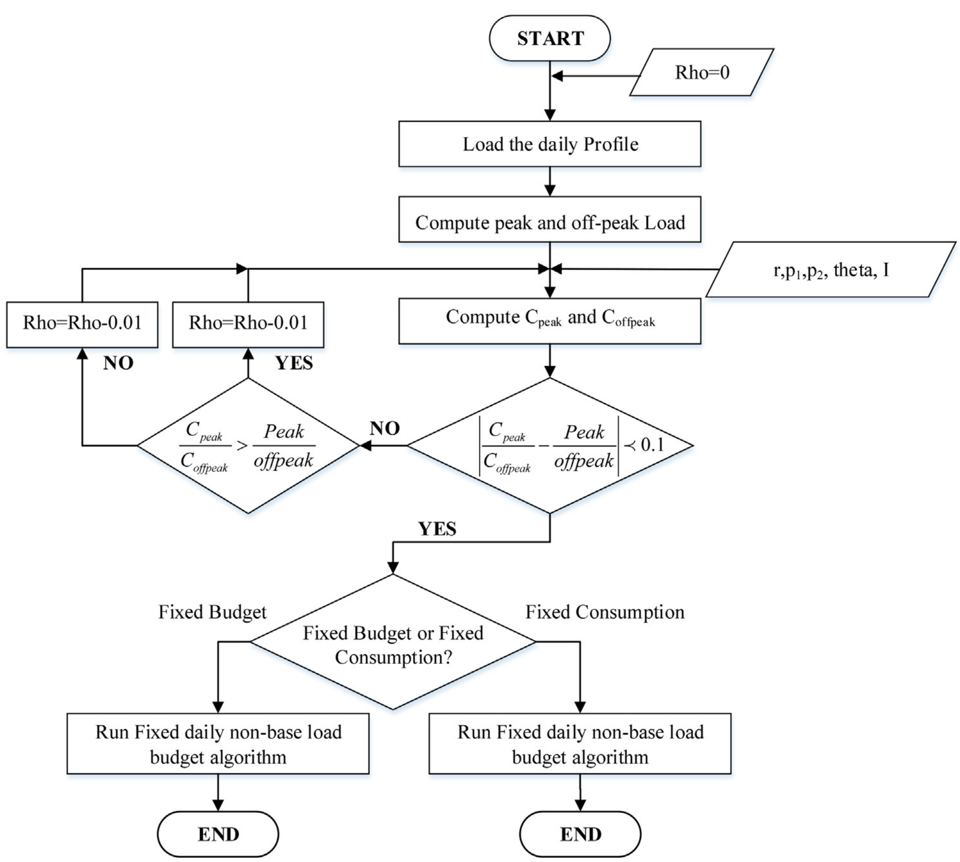

Fig. 8. Flow chart illustrating the model implementation.

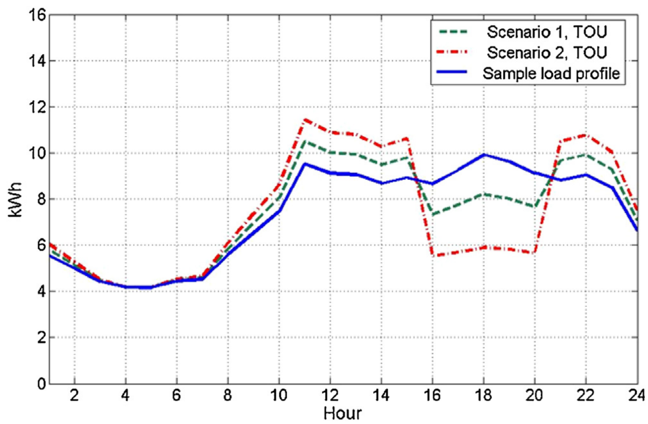

Fig. 10. Comparison between the different scenarios and flat rate load profile.

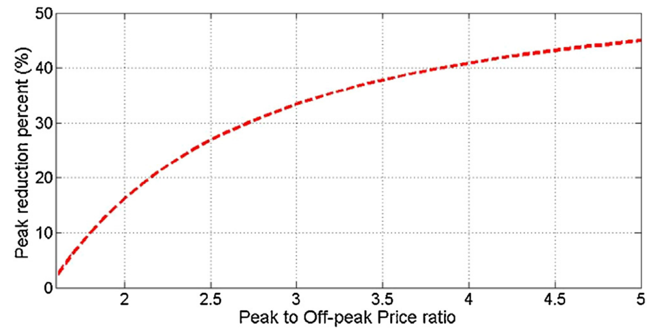

Fig. 11. Peak reduction vs. peak to off-peak price ratio.

different industrial loads and the residential customers. For the residential customers, it is acceptable to say that the night time load is almost inelastic. Therefore, in this paper, the minimum load at night is picked for the inelastic portion and is referred to "base-load" from now on. According to Fig. 9, 4.167 kW h (minimum load) is assumed to be the base-load.

In this scenario, it is assumed that every household has a hypothetical fixed budget for non-base-load daily consumption (elastic loads). As a result, each household tries to (a) shift the consumption cross-period (b) decrease the consumption to fit the budget constraint. Before offering the TOU pricing, the flat rate price is assumed to be 10 cents per kW h. The peak time in this study is marked between 16 and 20 (5 h) and the off-peak is marked to be what remains from the 24 h (19 h). The off-peak price is decreased to 8 cents per kW h, $p_{\text{off peak}}$ = \$0.08, the rate of the price reduction is consistent with the popular designs of the demand response pilot projects. Based on the results of the analysis of the consumption proportion, Fig. 7, $\rho = -0.46$ will give the same ratio of peak consumption to off-peak consumption. Assuming $\theta = 0.6$ (this value is chosen arbitrarily), Fig. 10 shows the results of the change in the load profile for scenario 1.

4.2.2. Scenario 2 (fixed non-base-load consumption)

In this scenario, it is assumed that every household has a fixed non-base-load daily consumption. Therefore, each household tries to just shift the consumption cross-period. The other assumptions are similar to the scenario 1. Fig. 10 shows the load profile for scenario 2 as well.

4.2.3. Discussion

Table 2 reports the peak and off-peak consumption calculated by the model. Scenarios 1 and 2 show 16.17% and 38.4% reduction in the peak consumption respectively. This decrease in peak consumption will shift to the off-peak consumption. Hence, scenarios 1 and 2 show 7.13% and 13.9%, rise in off-peak consumption.

As shown, both scenarios show reduction in the cost of the electricity which is consistent with the findings of the aforementioned pilot projects. Scenarios 1 and 2 show 1.64% and 7% reduction in the total cost respectively.

As discussed earlier, [35] found that the demand response expressed as a function of the peak reduction to the change in the peak to off-peak price ratio is nonlinear and is shaped in the form of an arc. A good model should be able to imitate the similar characteristics. Fig. 11 shows the price responsiveness of the proposed model for the price ratios of 1.5 to 5. The fixed non-base load budget algorithm is picked to extract the price responsiveness curve. The control for the substitution is same as scenario 1. The price responsiveness is sensitive to the control for the substitution. As it is shown, the figure is in the form of an arc. It saturates for the higher ratio of peak to off-peak price which can be explained by the law of diminishing marginal return in consumer theory.

With comparing the simulation results of Fig. 10 to the experimental results of [10] and the results of Fig. 11 with the results of reference [35], it could be claimed that the model successfully imitated the real life experience (i.e. the collection of multitudes of successful international pilot projects). Indeed, one of the advantages of the proposed model is that the proper control for the substitution can explain the demand response for all the price ratios unlike the methods based on the price elasticity which need to find the price elasticity for each price ratio.

Table 2
The consumption in each scenario.

Case	Peak consumption (kW h)	Off-peak consumption (kW h)	Daily budget (\$)
Flat rate ($r = 0$)	46.5	130	17.66
Scenario 1 ($r = 0$)	38.98	139.27	17.37
Scenario 2 ($r = 1$)	28.61	148.11	16.42

5. Conclusion

As digital and IT technologies mature in the power system area, utilities become more interested and empowered to introduce more complicated DR programs to the residential sector. Current DR models rely on many simplifications which do not provide enough flexibility dealing with more complex situations (e.g. residential customers). Therefore, the authors believe that it is necessary to examine the economics literature and use their latest findings to introduce a sophisticated model. As a result, this paper attempts to propose a new economic model to explain the customer's reaction to the change in the electricity price.

What distinguishes this model from the models discussed in the introduction section of this paper is twofold. First, this model has this capability to incorporate the willingness of customers to reallocate the consumption cross-periods. The lack of such capability is one of the shortcomings of the earlier models. In the absence of such capability, the model cannot be customized for each customer which is a major setback for such models when applied to residential customers. Second, this model reflects the classic economic decision making process of each individual. Therefore, it can easily be integrated into real time demand response models employed by utilities to (1) offer optimal dynamic pricing to customers and (2) select the optimum bidding strategy and planning.

Moreover, unlike the models discussed earlier in the paper, this model explains the underlying mechanism of the shift in the consumption and provides more intuitive sense about the decision making process of each customer.

To develop the model, first, the customer utility function was maximized. Then, the selection of the proper parameters for the model was discussed by sensitivity analysis. Next, the proposed algorithm and its implementation were elaborated. Afterwards, the numerical results of a case study was presented. To evaluate the performance of the model, the graph of peak to off-peak price ratio versus peak reduction percent was presented. It was understood that the graph fairly imitates the real observation results.

As a future study, we plan to incorporate behavioral characteristics of customers into the DR models, since this has been shown to have a significant impact on the customer's decision making [32,33]. In addition, while DR model proposed in this paper use only two time periods, upgrading it to incorporate multiple time periods which make the model more nuanced and flexible could be another interesting direction. This would be particularly important, in the future, when DR events are set to run almost every day. We also plan to utilize this model in a real time DR optimization to quantify its effect on the speed of convergence. In addition to the speed of convergence, the accuracy of model to predict the customers' response is another important aspect that needs to be assessed for a successful DR model. We plan to validate the accuracy of the proposed model on a pilot project as soon as the reliable data becomes available.

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