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Breakdown-free version of ILU factorization for nonsymmetric positive definite matrices

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ABSTRACT

In this paper a new ILU factorization preconditioner for solving large sparse linear systems by iterative methods is presented. The factorization which is based on A-biorthogonalization process is well defined for a general positive definite matrix. Numerical experiments illustrating the performance of the preconditioner are presented. A comparison with the well known preconditioner RIF$_p$ of Benzi and Tůma is also included.

1. Introduction

In this paper we consider the solution of linear systems of the form

$$Ax = b, \tag{1}$$

where the coefficient matrix $A \in \mathbb{R}^{n\times n}$ is large, sparse and nonsymmetric positive definite (NSPD), and b is a given right hand side vector using preconditioned conjugate gradient-type methods. Suppose that A admits the factorization

$$A = LDU, \tag{2}$$

where L, U^{T} are unit lower triangular matrices and D is a diagonal matrix. If $\bar{L}$ and $\bar{U}^{\mathrm{T}}$ are sparse unit lower triangular matrices approximating (in some sense) the matrices L and U^{T}, respectively, and $\bar{D}$ is a nonsingular diagonal matrix approximating D, then we say that matrix M with

$$M = \bar{L}\bar{D}\bar{U} \approx A, \tag{3}$$

is an incomplete LU (ILU) factorization preconditioner for matrix A. The transformed linear systems

$$AM^{-1}u = b, \quad M^{-1}u = x, \tag{4}$$

or

$$M^{-1}Ax = M^{-1}b, \tag{5}$$

have the same solution as system (1) and seem to be better-conditioned than the original system (1) to solve. It is well-known that an incomplete factorization of a general matrix A may fail due to the occurrence of zero pivots, regardless of

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