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COMPLETE PIVOTING STRATEGY FOR THE IUL PRECONDITIONER OBTAINED FROM BACKWARD FACTORED APPROXIMATE INVERSE PROCESS

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Abstract. In this paper, we use a complete pivoting strategy to compute the IUL preconditioner obtained as the by-product of the Backward Factored APproximate INVerse process. This pivoting is based on the complete pivoting strategy of the Backward IJK version of Gaussian Elimination process. There is a parameter α to control the complete pivoting process. We have studied the effect of different values of α on the quality of the IUL preconditioner. For the numerical experiments section, the IUL factorization which is coupled with the complete pivoting is compared to the ILUTP and to the left-looking version of RIF which is coupled with the complete pivoting strategy. As the preprocessing, we have applied the maximum weighted matching coupled with the Reverse Cuthill-Mckee (RCM) and multilevel nested dissection reordering.
Keywords: Backward factored APproximate INVerse, IUL preconditioner, backward IJK version of Gaussian elimination, complete pivoting, ILUTP, left-looking RIF with pivoting.
MSC(2010): Primary: 65F10 ; Secondary: 65F50, 65F08.

1. Introduction

One can use the explicit and implicit preconditioner M for the linear system of equations of the form

$$Ax = b, \tag{1.1}$$

where the coefficient matrix $A \in \mathbb{R}^{n\times n}$ is nonsingular, large, sparse and nonsymmetric and also $x, b \in \mathbb{R}^n$. An explicit preconditioner M for system (1.1) is an approximation of the matrix A^{-1}. We can use this preconditioner to change the original system (1.1) to the right or left preconditioned systems and then, solve the preconditioned system by one of the Krylov subspace methods [17].

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In this case, we only need matrix-vector products which is really suitable for parallel architecture.

In 1993, Luo presented the **B**ackward **F**actored **INV**erse or BFINV algorithm which computes the inverse factorization of A in the form of

$$A^{-1} = \bar{Z}\bar{D}^{-1}\bar{W}, \tag{1.2}$$

where $\bar{W}$ and $\bar{Z}^T$ are unit upper triangular matrices and $\bar{D}$ is a diagonal matrix [11]. By applying a dropping rule for the entries of the $\bar{W}$ and $\bar{Z}$ matrices in the BFINV algorithm, the explicit preconditioner M is computed as

$$A^{-1} \approx M = ZD^{-1}W, \tag{1.3}$$

where $W \approx \bar{W}$, $D \approx \bar{D}$, $Z \approx \bar{Z}$ and the process is termed as the **B**ackward **F**actored **AP**proximate **INV**erse or BFAPINV. The implementation details to compute this explicit preconditioner can be found in [23].

In 1999, Zhang presented the **F**orward **F**actored **INV**erse or FFINV algorithm which computes the factorization (1.2). In this case, matrices $\bar{Z}$ and $\bar{W}$ are unit upper and unit lower triangular, respectively and $\bar{D}$ is again a diagonal matrix [22]. Using a dropping rule in this algorithm will compute the explicit preconditioner (1.3) and the process is termed as the **F**orward **F**actored **AP**proximate **INV**erse or FFAPINV [13].

In [20], the authors could find a relation between the FFINV algorithm and the left-looking version of the A-biconjugation process of Benzi and Tůma [1]. Based on this relation they showed that the explicit preconditioner (1.3) which is computed from the FFAPINV algorithm is exactly the left-looking version of the AINV preconditioner.

An implicit preconditioner for the system (1.1) is an approximation of matrix A. This preconditioner can also be used as the right or left preconditioner. When using the Krylov subspace methods to solve this preconditioned system, we face the forward and backward solving which are the bottle necks in the parallel implementation of implicit preconditioners in recent years. Solving such a problem is so crutial to apply an implicit preconditioner on parallel machines [9]. In [13], we could compute an implicit preconditioner M as the by-product of the BFAPINV process. This preconditioner is in the form of

$$A \approx M = UDL, \tag{1.4}$$

where U and L^T are unit upper triangular matrices and D is a diagonal matrix. This preconditioner is an incomplete UDL factorization. We have merged the factors D and L of this factorization and then, have termed it as the IULBF. This notation refers to the IUL factorization obtained from **B**ackward **F**actored approximate inverse process. In the factorizations (1.3) and (1.4), $L^{-1} \approx Z$ and $U^{-1} \approx W$.